Smoothness via directional smoothness and Marchaud's theorem

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The talk is based on a recent joint work with M. Johanis (J. Math. Anal Appl. 2015, 594-607).

Classical Marchaud's theorem (1927) asserts that if f is a bounded function on $[a, b]$, $k \in \mathbb{N}$, and the $(k+1)$th modulus of smoothness $\omega_{k+1}(f;t)$ is so small that $\eta(t) = \int_0^t \frac{\omega_{k+1}(f;s)}{s^{k+1}}\, ds < +\infty$ for $t > 0$, then $f \in C^k\big((a, b)\big)$ and $f^{(k)}$ is uniformly continuous with modulus $c\eta$ for some $c > 0$. Using a known version of the converse of Taylor theorem we easily deduce Marchaud's theorem for functions on certain open connected subsets of Banach spaces from the classical one-dimensional version. In the case of a bounded subset of $\mathbb{R}^n$ our result is more general than that of H. Johnen and K. Scherer (1973), which was proved by quite a different method.