Mechanical Analogs For One Dimensional Viscoelastic Models.

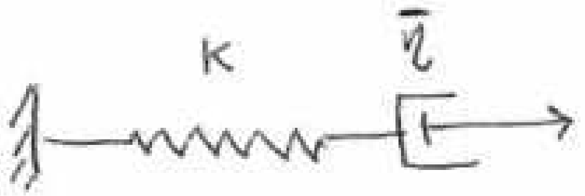


Figure 1 : Maxwell Fluid.

If the spring is a linear spring, the model is a one dimensional linearized Maxwell fluid.

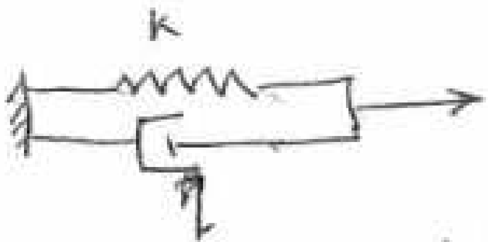


Figure 2 : Kelvin-Voigt solid

If the spring is a linear spring, the model is a linearized Kelvin-Voigt solid.

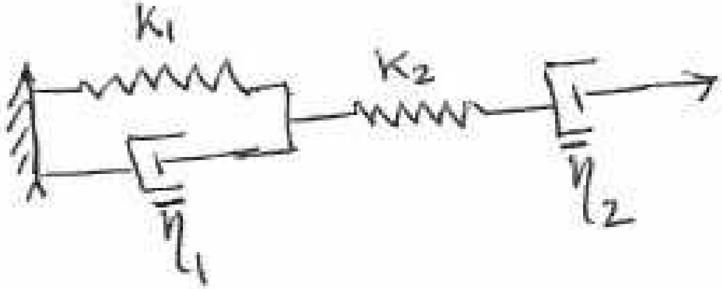
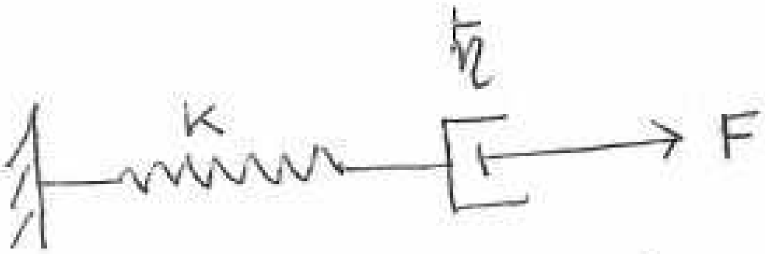


Figure 3: One of five arrangements of springs and dashpots that leads to a Burgers' fluid model. Includes the Maxwell and Oldroyd-B model as special cases. (See Karra and Rajagopal (2009)).

One dimensional Maxwell fluid model derivation based on the mechanical analog.



Let Δ denote the elongation

Let $\cdot$ denote time derivative.

Then total elongation $\Delta = \underbrace{\Delta_s}_{\text{elongation in the spring}} + \underbrace{\Delta_d}_{\text{elongation in the dashpot}}$. (1)

Equilibrium $\Rightarrow$ $F_s = F_d = F$ (2).

F_s - force in the spring

F_d - force in the dashpot

Constitutive Equations :

For the spring : $F_s = K\Delta_s$. (3)

For the dashpot : $F_d = \bar{\eta}\,\dot{\Delta}_d$. (4)

$$(1)-(4) \Rightarrow \dot{\Delta}_s + \dot{\Delta}_d = \frac{\dot{F}_s}{K} + \frac{F_d}{\bar{\eta}} = \frac{\dot{F}}{K} + \frac{F}{\bar{\eta}} = \dot{\Delta}$$

In terms of stress-strain this leads to

$$\boxed{\sigma + \lambda\dot{\sigma} = \eta\dot{\varepsilon}}$$

Units of $\bar{\eta}$ and η are different.

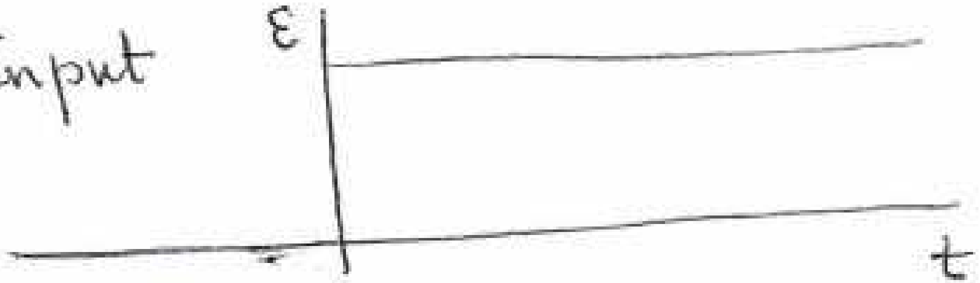
$\dim(\lambda) = T$ (time)

$\dim(\eta) = ML^{-1}T^{-1}$

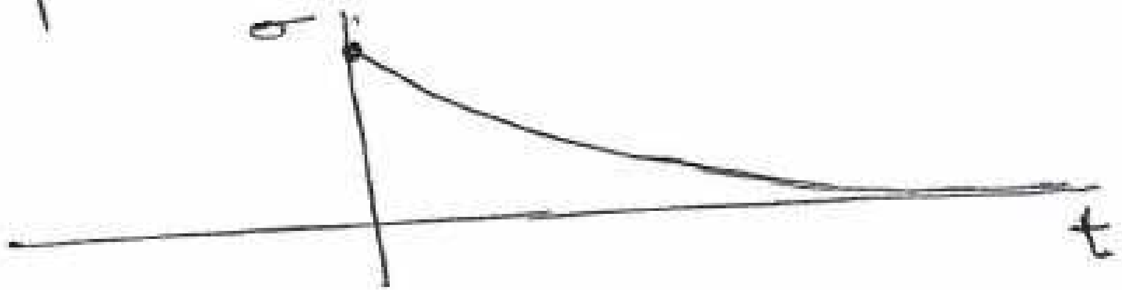
Maxwell Fluid.

ε – strain

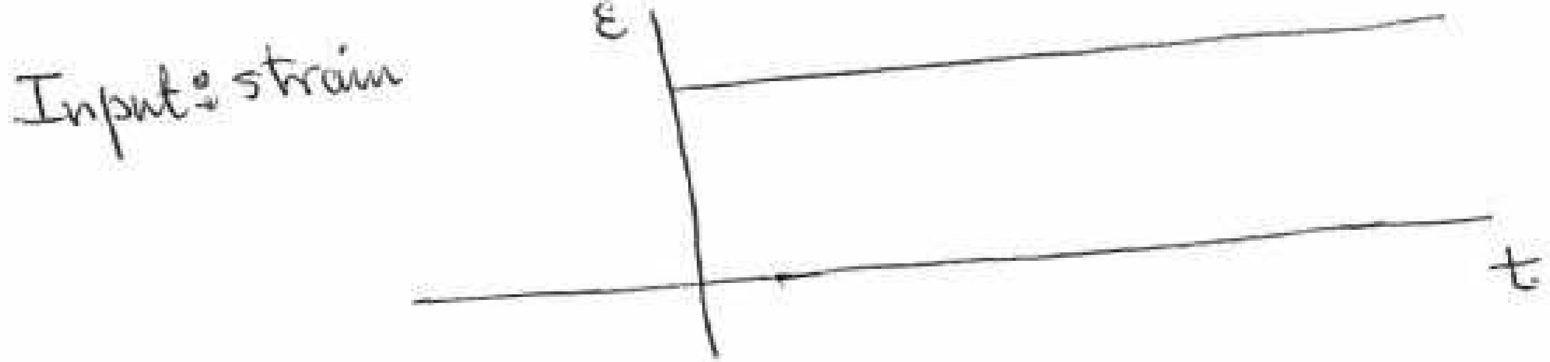
Input

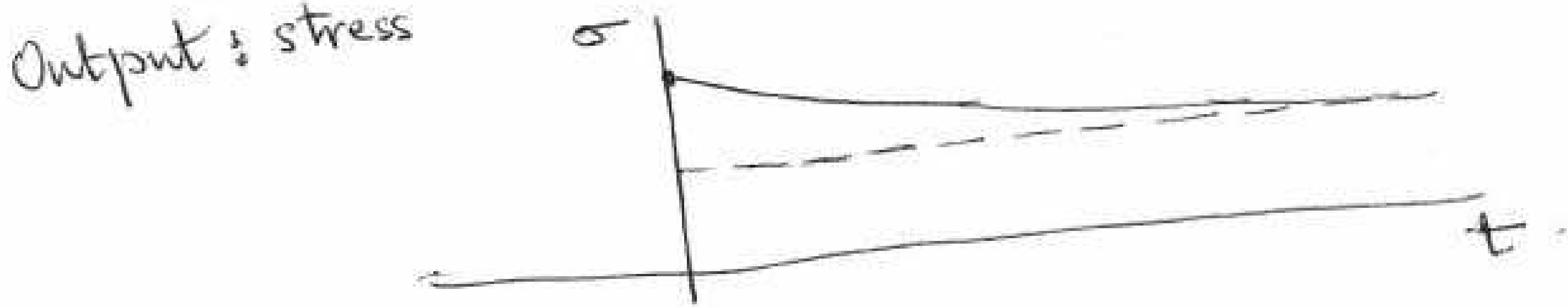


Output stress.



Kelvin-Voigt Solid.





The above is called a stress relaxation test.

- Helmholtz Potential Approach: Will not always work.
- Pick rate of entropy production
    1. due to conduction
    2. due to work being converted into energy in thermal form
    3. due to mixing
    4. due to phase transition
    5. due to growth

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- Part of the energy supplied can
  1. change the kinetic energy
  2. change the potential energy
  3. change the "strain" or "stored" energy
  4. etc.
- Need to know how the energy that is stored can be recovered
  1. in a purely mechanical process
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- Restrict ourselves to isotropic processes.

$$\xi = \mathbf{T}.\mathbf{D} - \rho\dot{\psi} \tag{20}$$

ψ – Helmholtz potential

- If the material is incompressible, then we need:

$$\text{div}(\mathbf{v}) = 0. \tag{21}$$

- Maximize ξ subject to (20) and (21) as constraints.
- Auxiliary function to be maximized

$$\Phi = \xi + \lambda_1 \left(\xi - \mathbf{T}.\mathbf{D} - \rho\dot{\psi} \right) + \lambda_2(\text{div}(\mathbf{v})). \tag{22}$$

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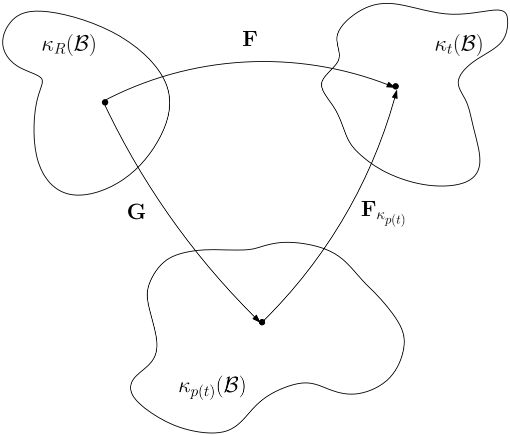

- $\mathbf{B}_{\kappa_{p(t)}} := \mathbf{F}_{\kappa_{p(t)}} \mathbf{F}^{\top}_{\kappa_{p(t)}}$
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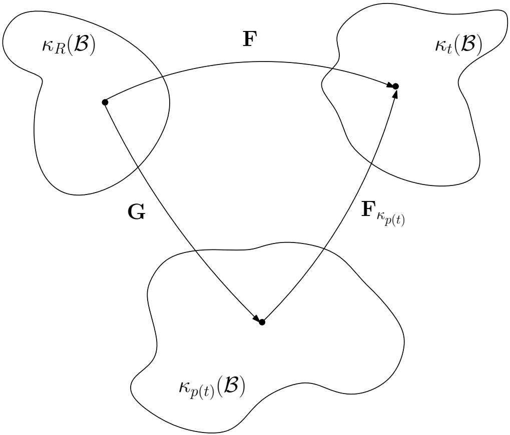

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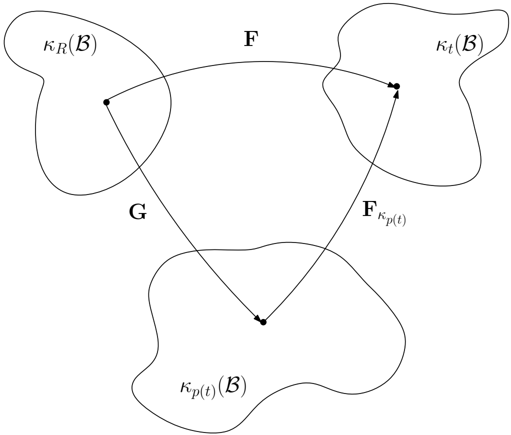

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  Upper convected Oldroyd derivative
- $\overset{\nabla}{\mathbf{B}}_{\kappa_{p(t)}} = -2\mathbf{F}_{\kappa_{p(t)}}\mathbf{D}_{\kappa_{p(t)}}\mathbf{F}^{\mathsf{T}}_{\kappa_{p(t)}}$
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- Leads to a generalization of a Maxwell fluid.
- $\mathbf{T} = -p\mathbf{I} + \mu \mathbf{B}_{\kappa_{p(t)}}, \quad \overset{\nabla}{\mathbf{B}}_{\kappa_{p(t)}} = -\frac{2\mu}{\eta}\left[\mathbf{B}_{\kappa_{p(t)}} - \lambda \mathbf{I}\right]$
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- Get a generalization of the Oldroyd-B model.
- Can develop generalization of Burgers' fluid, etc.
- Can develop generalization of any system of springs and dashpots.
- Can obtain models which use ideas such as "confirmation tensors": FENE-P model.
- Can develop models for anisotropic fluids.
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- Pick $\xi = \eta \mathbf{D}_{\kappa_{p(t)}} . \mathbf{B}_{\kappa_{p(t)}} \mathbf{D}_{\kappa_{p(t)}} + \eta_1 \mathbf{D}.\mathbf{D}$
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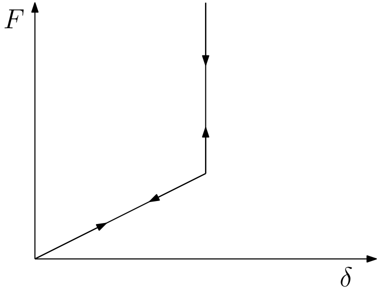

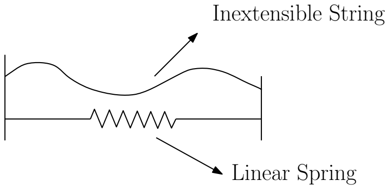

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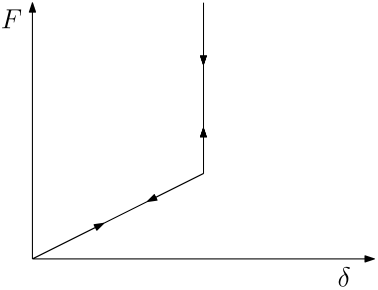

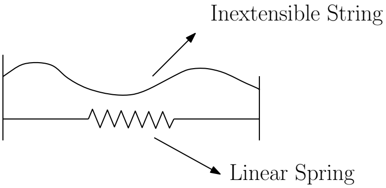

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- Ideas developed by D'Alembert and Bernoulli and used by Lagrange that appeal to constraint forces doing no work are **WRONG**. This is at the heart of how one shows that

$$\mathbf{T} = -p\mathbf{I} + \mathbf{S}$$

in an incompressible fluid.

Gibbs Potential Approach

- $G = G(\mathbf{S}), \quad \mathbf{S} = \dfrac{\mathbf{T}}{\rho}$.
- $\xi = \xi\left(\mathbf{S}, \rho, \mathbf{D}\right)$
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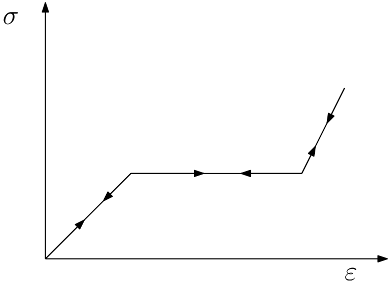

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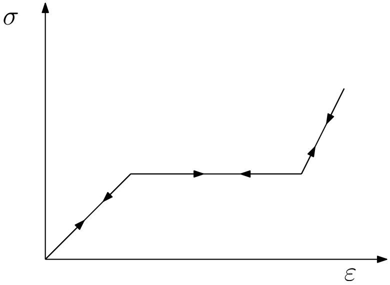

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