New Perspectives in Modeling Non-Newtonian Fluids

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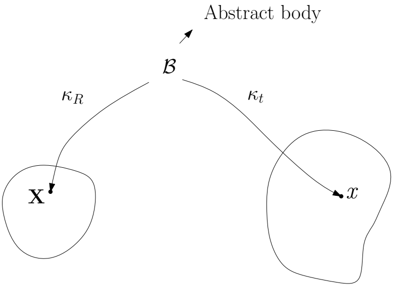

- κ_R, κ_t – placers
- $\kappa_R(\mathcal{B})$, $\kappa_t(\mathcal{B})$ – configurations
- Motion: One-parameter family of placers.

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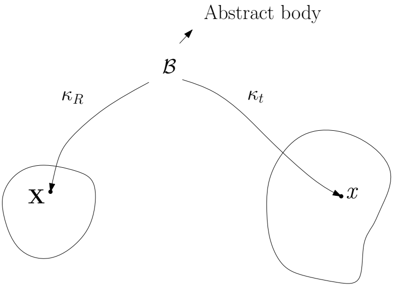

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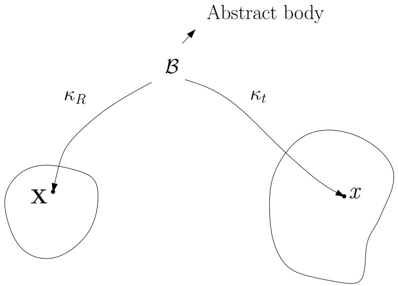

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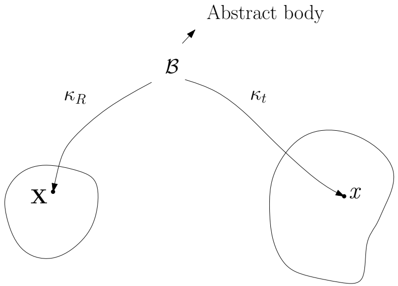

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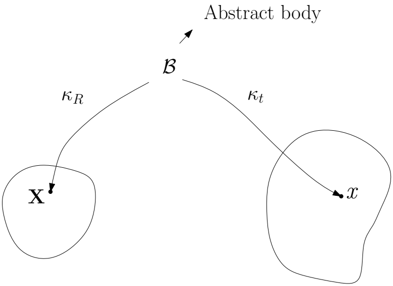

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- Relative Motion

$$\xi := \chi_{\kappa_R}\left(\chi_{\kappa_R}^{-1}(\boldsymbol{x}, t), \tau\right) = \chi_t\left(\boldsymbol{x}, \tau\right). \tag{2}$$

- Deformation Gradient

$$\mathbf{F}_{\kappa_R} := \frac{\partial \chi_{\kappa_R}}{\partial \mathbf{X}} \tag{3}$$

(An unnecessary and ill-conceived concept).

- $\mathbf{F}_{\kappa_R}$ is a linear transformation from the tangent space at $\mathbf{X}$ to the tangent space at $\boldsymbol{x}$.
- Relative Deformation Gradient

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Classical Constitutive Relations

- Classical Cauchy Elasticity

$$\mathbf{T} = \mathbf{f}_{\kappa_R}\left(\mathbf{F}_{\kappa_R}\right). \tag{9}$$

- Classical Navier Stokes Fluid

$$\mathbf{T} = -p(\rho_{\kappa_t})\mathbf{I} + \lambda(\rho_{\kappa_t})\left(\mathrm{tr}\mathbf{D}_{\kappa_t}\right)\mathbf{I} + 2\mu\mathbf{D}_{\kappa_t}, \tag{10}$$

$$\mathbf{D}_{\kappa_t} = \frac{1}{2}\left[\nabla_x \mathbf{v} + (\nabla_x \mathbf{v})^{\mathsf{T}}\right]$$

- ρ_{κ_t} – density, p – thermodynamic pressure
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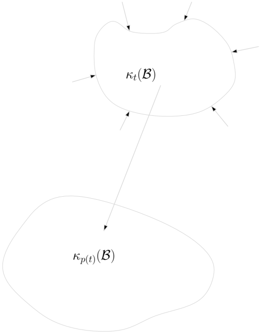
- "Natural Configuration" – The configuration the body takes on the removal of the external stimuli.



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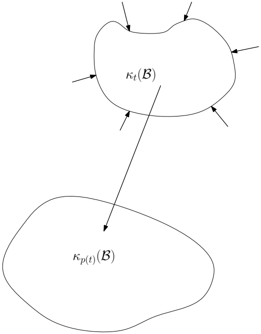

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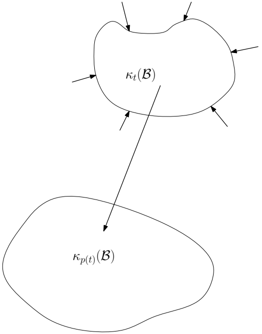

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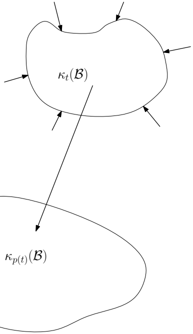

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Notion of "Natural Configuration"

- A body is not necessarily a fixed set of particles – Growth and Adaptation of biological bodies.
- To define a "Body", it is necessary to know the natural configurations a body is capable of existing in. In any process, we need to know which natural configurations are accessed.
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Notion of "Natural Configuration"

- The same piece of steel can undergo
  - a non-dissipative process
  - twinning
  - slip
  - solid to solid phase transformation
  - melting
  - etc.
- We need to define "states", "processes", "process classes" : Isothermal, Adiabatic, Isentropic, Isenthalpic, Isobaric, Isotonic, etc.
- Different natural configurations are accessed during different processes. The "natural configuration" is a part of the specification of the "state" of the body.

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- If one inhomogeneously deforms a body and then removes the traction, it is possible that the unloaded body will not fit together compatibly and be simultaneously stress free in an Euclidean Space.
- However, it can be unloaded to fit together in a non-Euclidean space in which it is stress-free (Eckart - 1940s).
- A “sufficiently small” neighborhood of a material point can be unloaded to a stress-free configuration. Here, “sufficiently small” corresponds to a neighborhood in which the deformation is “essentially homogeneous”.
- Henceforth, we will assume, for the sake of illustration the deformation is homogeneous.

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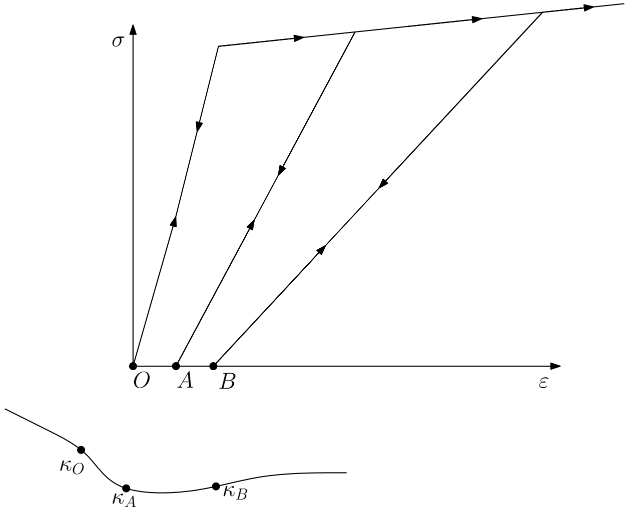
Natural Configuration in Traditional Plasticity



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- A body can have a finite number of natural configurations.
    - Twinning, Solid to Solid phase transition
- A body can have infinity of natural configurations.
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Balance Laws

- Balance of Mass:

$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \mathbf{v}) = 0. \tag{15}$$

- Balance of Linear Momentum:

$$\rho \frac{d\mathbf{v}}{dt} = \operatorname{div}\left(\mathbf{T}^{\mathsf{T}}\right) + \rho \mathbf{b}. \tag{16}$$

- Balance of Angular Momentum in the absence of body couples:

$$\mathbf{T} = \mathbf{T}^{\mathsf{T}}. \tag{17}$$

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ϵ – specific internal energy
$\mathbf{q}$ – heat flux vector
r – radiant heating
$\mathbf{L} = \nabla_x \mathbf{v}$.

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- The evolution of the natural configuration, amongst other things, is determined by the maximization of entropy production.
- Ziegler suggested maximization of the rate of entropy production, not within the context of determining allowable processes. Also, he made mathematical errors. He also did not consider implicit equations wherein the maximization is with respect to the stress.
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