Kinetic models for dilute polymers: analysis, approximation and computation

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We review recent analytical and computational results for macroscopic-microscopic bead-spring models that arise from the kinetic theory of dilute solutions of incompressible polymeric fluids with noninteracting polymer chains, involving the coupling of the unsteady Navier–Stokes system in a bounded domain $\Omega \subset \mathbb{R}^d$, $d = 2$ or 3, with an elastic extra-stress tensor as right-hand side in the momentum equation, and a (possibly degenerate) Fokker–Planck equation over the $(2d+1)$-dimensional region $\Omega \times D \times [0, T]$, where $D \subset \mathbb{R}^d$ is the configuration domain and $[0, T]$ is the temporal domain. The Fokker–Planck equation arises from a system of (Itô) stochastic differential equations, which models the evolution of a $2d$-component vectorial stochastic process comprised by the d-component centre-of-mass vector and the d-component orientation (or configuration) vector of the polymer chain. We show the existence of global-in-time weak solutions to the coupled Navier–Stokes–Fokker–Planck system for a general class of spring potentials including, in particular, the widely used finitely extensible nonlinear elastic (FENE) potential. The numerical approximation of this high-dimensional coupled system is a formidable computational challenge, complicated by the fact that for practically relevant spring potentials, such as the FENE potential, the drift term in the Fokker–Planck equation is unbounded on ∂D. We shall present numerical simulations for this coupled high-dimensional micro-macro model and consider the analysis of the numerical algorithms. The work presented in this lecture series is based on joint research with John W. Barrett (Imperial College London) and David Knezevic (Massachusetts Institute of Technology).