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Research Team 1
Mathematical Institute of the Charles University
Sokolovská 83, 186 75 Praha 8
http://ncmm.karlin.mff.cuni.cz/

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BEM implementation of energetic solutions for quasistatic delamination problems

C. G. Panagiotopoulos[1], V. Mantič[1], T. Roubíček[2,3]

[1] Group of Elasticity and Strength of Materials, Department of Continuum Mechanics, School of Engineering, University of Seville, Seville, ES-41092, Spain
[2] Mathematical Institute, Charles University, Czech Republic, Prague, CZ-18675
[3] Institute of Thermodynamics of the ASCR, Czech Republic, Prague, CZ-18200
mantic@esi.us.es, cpanagiotopoulos@us.es, tomas.roubicek@mff.cuni.cz

Abstract

The problem of the onset and propagation of an elastic-brittle delamination is considered. We study delamination processes for elastic bodies glued by an adhesive to each other or to a rigid outer boundary. The interfacial adhesive is assumed to store and also dissipate a specific amount of energy during the delamination process. Damage along the interfaces is taken into account by introducing an interface damage variable. The present approach is based on the so-called energetic-solution concept. After introducing an implicit time discretization and a spatial discretization along the boundaries, the boundary element method (BEM) is utilized to solve the pertinent recursive boundary-value problems arising at each time step and compute the stored elastic energy. The whole solution process is based on the global minimization of the sum of the elastic potential energy in the solids and adhesive layer, defined in terms of the displacements along boundaries and the damage parameter along the interfaces, and of the dissipated energy at each time step. Numerical solution of a delamination problem is presented to demonstrate the capabilities of this new approach.

Keywords: Interface fracture, rate-independent quasistatic model, adhesive contact, energetic solutions, delamination, debonding, interface damage, Boundary Element Method.

1 Introduction

A basic model for quasistatic delamination, proposed by Frémond [1, 2], involves a damage-type variable, called a delamination parameter and denoted by z in what follows, reflecting the destruction of the bonds in the a-priori given delamination surfaces or part of outer boundary. This basic model incorporates merely the Griffith concept [3], namely the philosophy that the crack grows as soon as the so-called energy release rate during the delamination propagation is greater or equal than the phenomenologically prescribed activation energy (per unit area of a new crack surface), called also fracture energy. This activation energy is, in fact, equal to the dissipated energy. As such, the energy needed (and disipated) for the delamination is assumed to be *rate-independent* (after neglecting all inertial/visco/thermo effects), which is reflected in the *quasistatic* model. The weak surface undergoing delamination can be considered either *purely brittle* or *elastic-brittle*, reflecting that the adhesive gluing this weak surface has a certain elastic response. The conctact is assumed unilateral and, for simplicity, frictionless and the delamination is assumed mode insenstive. This approach was first developed in [4]–[8], cf. also [11, Chap.14].

The goal of this work is to present briefly, from the theoretical and numerical viewpoint, some basic features of the present approach. As the damage processes occurs only on boundaries, the spatial discretisation can advantageously be made by the BEM. The implementation of the collocation BEM [9, 10] is described first. Then, the computational procedure is validated by 2-dimensional simulation of a delamination process, showing that it can really produce the expected responses. For the sake of brevity, implementation of the above methodology will only briefly presented, full details as well as further numerical simulations and discussion are incorporated into a forthcoming paper [12]. A more thorough theoretical presentation of such models may be found in [13], where also a finite-element method (FEM) implementation and numerical simulations are presented.

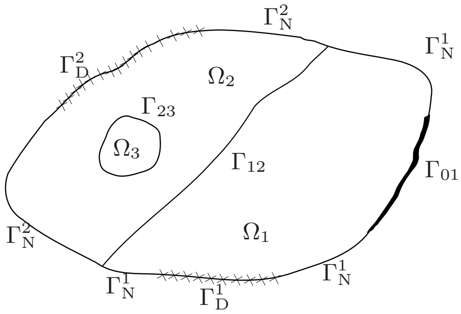


Figure 1: Schematic illustration of the geometry and of the notation for 2-dimensional case of 3 mutually contacted subdomains, i.e. d=2 and N=3.

2 Theoretical background, quasistatic rate-independent evolution, delamination model

We will consider the evolution on a fixed finite time interval $[0,T]$ governed by a *stored energy* functional $\mathscr{E} : [0,T] \times \mathscr{U} \times \mathscr{Z} \to \mathbb{R} \cup \{\infty\}$ and a *dissipated energy* functional $\mathscr{R} : \mathscr{X} \to [0,\infty]$, where $\mathscr{U}$ stands for the linear space of displacement field u, $\mathscr{Z}$ for that of damage parameter field z while $\mathscr{X}$ for the linear space of time derivatives of z. The rate-independent evolution we have in mind is governed by the following system of *doubly nonlinear* degenerate *parabolic/elliptic variational inclusions*:

$$\partial_u \mathscr{E}(t,u,z) \ni 0 \quad \text{and} \quad \partial\mathscr{R}\big(\dot{z}\big) + \partial_z \mathscr{E}(t,u,z) \ni 0, \tag{1}$$

where $\dot{z} := \frac{\mathrm{d}z}{\mathrm{d}t}$ and the symbol "∂" refers to a (partial) subdifferential, relying on that $\mathscr{R}(\cdot)$, $\mathscr{E}(t,\cdot,z)$, and $\mathscr{E}(t,u,\cdot)$ are convex functionals. Let us recall that the subdifferential $\partial f(x)$ of a convex function $f : X \to \mathbb{R} \cup \{\infty\}$ at a point x is defined as the convex closed subset $\partial f(x) := \{x^*{\in}X^*;\ \forall v{\in}X :\ f(x) + \langle x^*, v{-}x\rangle \leq f(v)\}$ of the dual space X^*. Let $\Omega \subset \mathbb{R}^d$ $(d = 2,3)$ be a bounded Lipschitz domain, and let us consider its decomposition into a finite number of mutually disjoint Lipschitz subdomains Ω_i, $i = 1,...,N$. We denote formally $\Omega_0 := \mathbb{R}^d \setminus \bar{\Omega}$. Further denote $\Gamma_{ij} = \partial\Omega_i \cap \partial\Omega_j$ the (possibly empty) boundary between Ω_i and Ω_j, $i,j = 1,...,N$. For $i,j = 1,...,N$, Γ_{ij} represents a prescribed $(d{-}1)$-dimensional surface, which may undergo delamination (also called debonding). We also consider a possible debonding on some parts of the outer boundary $\Gamma_{0i} \subset (\partial\Omega \cap \bar{\Omega}_i)$. We assume that the rest of the outer boundary $\partial\Omega$, i.e. $\partial\Omega \setminus \cup_{i=1}^N \Gamma_{0i}$, is the union of two disjoint subsets $\Gamma_{\rm D}$ of a non zero measure and $\Gamma_{\rm N}$. On the Dirichlet part of the boundary $\Gamma_{\rm D}$ and on $\cup_{i=1}^N \Gamma_{0i}$ we impose a time-dependent boundary displacement $w_{\rm D}(t)$, while Neumann part of the boundary $\Gamma_{\rm N}$ is considered free. Therefore, any admissible displacement $u : \cup_{i=1}^N \Omega_i \to \mathbb{R}^d$ has to be equal to a prescribed "hard-device" loading $w_{\rm D}(t)$ on $\Gamma_{\rm D}$ and $(u{-}w_{\rm D}(t)){\cdot}\nu \geq 0$ on $\cup_{i=1}^N \Gamma_{0i}$. Delamination (or debonding), can be developed on the surface

$$\Gamma_{\rm C} := \bigcup_{0\leq i<j\leq N} \Gamma_{ij}, \tag{2}$$

where phenomenological elastic constant $\kappa_{\rm n}$ and $\kappa_{\rm t}$ represent the stiffnesses of a thin linear elastical adhesive layer in the normal and tangential directions, respectively. Usually $\kappa_{\rm n}$ is greater than $\kappa_{\rm t}$, for isotropic adhesive a condition $\kappa_{\rm n}/\kappa_{\rm t} \geq 2$ has been deduced in [14] (see also further references therein).

On each Linear elastic subdomain Ω_i, we consider linear homogeneous, possibly anisotropic material with the elastic-moduli tensor $\mathbb{C}^{(i)}$. Here we used the notation $e(u) = \frac{(}{\nabla}u)^\top + \frac{1}{2}\nabla u$ for the small strain tensor, $[\![u]\!]_{\rm n} = [\![u]\!]{\cdot}\nu$ with ν a unit normal to $\Gamma_{\rm C}$, and $[\![u]\!]$ the differences of traces from both sides of $\Gamma_{\rm C}$. On Γ_{0i}, we used the convention that $w_{\rm D}$ stands as the prescribed trace from $\Omega_0 := \mathbb{R}^d \setminus \bar{\Omega}$. Also $[\![u]\!] = [\![u]\!]_{\rm n}\nu + [\![u]\!]_{\rm t}$

with $[\![u]\!]_\mathrm{n} = [\![u]\!]\cdot\nu$ with ν a unit normal to Γ_C. Considering $u_\mathrm{D} = u_\mathrm{D}(t)$ and extension of $w_\mathrm{D}(t)$ on Ω and the shifted displacement $u + u_\mathrm{D}$ in place of u, the functionals occuring in (1) are considered as:

$$\mathscr{E}(t,u,z) := \begin{cases} \displaystyle\sum_{i=1}^{N}\int_{\Omega_i}\frac{1}{2}\mathbb{C}^{(i)}e\big(u-u_\mathrm{D}(t)\big):e\big(u-u_\mathrm{D}(t)\big)\,\mathrm{d}x & \\ \displaystyle +\int_{\Gamma_\mathrm{C}} z\Big(\frac{\kappa_\mathrm{n}}{2}\big|[\![u]\!]_\mathrm{n}\big|^2+\frac{\kappa_\mathrm{t}}{2}\big|[\![u]\!]_\mathrm{t}\big|^2\Big)\,\mathrm{d}S & \text{if } [\![u]\!]_\mathrm{n}\geq 0,\ 0\leq z\leq 1 \text{ on } \Gamma_\mathrm{C}, \\ & \text{and } \ u=0 \ \text{ on } \Gamma_\mathrm{D}, \\ \infty & \text{elsewhere,} \end{cases} \tag{3a}$$

$$\mathscr{R}(\dot z) = \mathscr{R}(\dot z) := \begin{cases} \displaystyle\int_{\Gamma_\mathrm{C}} a_\mathrm{I}|\dot z|\mathrm{d}S & \text{if } \dot z \leq 0 \text{ a.e. on } \Gamma_\mathrm{C}, \\ \infty & \text{otherwise.} \end{cases} \tag{3b}$$

In case $d = 3$, the physical dimensions are: $[a_\mathrm{I}]$=J/m^2 and $[\kappa_\mathrm{t}] = [\kappa_\mathrm{n}]$ =J/m^4=N/m^3. From (1), one can identify that the activation criterion to trigger the delamination is given

$$\frac{1}{2}\Big(\kappa_\mathrm{n}\big|[\![u]\!]_\mathrm{n}\big|^2+\kappa_\mathrm{t}\big|[\![u]\!]_\mathrm{t}\big|^2\Big) \leq a_\mathrm{I} \tag{4}$$

and a_I plays the role of the prescribed fracture energy.

Note that almost the same model, after some minor modification, may be established in order to degenerate to the *purely brittle* case as shown in [8, 13] for elastic constants κ_n and κ_t tending to infinite values.

We use the notation $\mathrm{B}([0,T];\mathscr{U})$ for the Banach space of bounded measurable functions $[0,T] \to \mathscr{U}$ defined everywhere, and $\mathrm{BV}([0,T];\mathscr{X})$ for functions $[0,T] \to \mathscr{X}$ with bounded variation. Recall that the variation of $z : [0,T] \to \mathscr{X}$ is defined as $\sup\sum_{j=1}^{N}\|z(t_j){-}z(t_{j-1})\|$, where $\|\cdot\|$ is the norm on $\mathscr{X}$ and the supremum is taken over all partitions of $[0,T]$. Moreover, $\mathscr{R}$ is assumed coercive on the Banach space in the sense that $\mathscr{R}(z) \geq \varepsilon\|z\|$ for some $\varepsilon > 0$.

A fruitful concept of a certain weak solution to the doubly nonlinear inclusion with degree-1 homogenous dissipation potential $\mathscr{R}$, called energetic solutions, was developed by Mielke et al. [15, 16], cf. also [17] for a survey. In the convex case, this concept is essentially equivalent to conventional weak-solution concept, while in our case where $\mathscr{E}(t,\cdot,\cdot)$ is nonconvex this concept represents a generalization which is well amenable to mathematical analysis and numerical implementation and applicable to engineering problems, too.

Definition 2.1 (Energetic solutions, [15, 16]) *The process* $(u,z) : [0,T] \to \mathscr{U}\times\mathscr{Z}$ *is called an energetic solution to the initial-value problem* (1) *given by* $(\mathscr{U}\times\mathscr{Z},\mathscr{E},\mathscr{R})$ *and the initial condition* (u_0,z_0) *if* $u \in \mathrm{B}([0,T];\mathscr{U})$, $z \in \mathrm{B}([0,T];\mathscr{Z})\cap\mathrm{BV}([0,T];\mathscr{X})$, *and*

(i) the energy equality holds:

$$\underbrace{\mathscr{E}(T,u(T),z(T))}_{\substack{\text{stored energy}\\ \text{at time } t=T}} + \underbrace{\mathrm{Diss}_\mathcal{R}(z,[0,T])}_{\substack{\text{energy dissipated}\\ \text{during } [0,T]}} = \underbrace{\int_0^T \mathscr{E}_t'(t,u,z)\,\mathrm{d}t}_{\substack{\text{work done by}\\ \text{mechanical load}}} + \underbrace{\mathscr{E}(0,u_0,z_0)}_{\substack{\text{stored energy}\\ \text{at time } t=0}}. \tag{5}$$

where

$$\mathrm{Diss}_\mathcal{R}(z,[0,T]) := \sup\sum_{j=1}^{N}\mathcal{R}(z(t_j)-z(t_{j-1})), \tag{6}$$

with the supremum taken over all partitions $0 \leq t_0 < t_1 < ... < t_{N-1} \leq t_N \leq T$.

(ii) the following stability inequality holds for any $t \in [0,T]$*:*

$$\forall(\tilde u,\tilde z) \in \mathscr{U}\times\mathscr{Z}: \quad \mathscr{E}(t,u,z) \leq \mathscr{E}(t,\tilde u,\tilde z) + \mathscr{R}(\tilde z{-}z), \tag{7}$$

(iii) the initial conditions $u(0) = u_0$ *and* $z(0) = z_0$ *hold.*

Here $\mathscr{U} = W^{1,2}(\Omega\backslash\Gamma_\mathrm{C};\mathbb{R}^d)$, $\mathscr{X} := L^1(\Gamma_\mathrm{C})$, and $\mathscr{Z} := L^\infty(\Gamma_\mathrm{C})$. If w_D has an extension u_D belonging to $C^1([0,T];W^{1,2}(\Omega;\mathbb{R}^d)$ and the initial condition (u_0,z_0) is stable in the sense that $\mathscr{E}(0,u_0,z_0) \leq \mathscr{E}(0,\tilde u,\tilde z)+\mathscr{R}(\tilde z{-}z_0)$ for all $(\tilde u,\tilde z) \in \mathcal{U}\times\mathcal{Z}$, one can show that the energetic solutions do exist.

3 Discretisation and numerical implementation

We make an implicit time discretisation by using, for simplicity, an equidistant partition of $[0, T]$ with a time-step $\tau > 0$, assuming $T/\tau \in \mathbb{N}$. This leads to a recursive minimization problem:

$$\left.\begin{array}{ll}\text{minimize} & (u, z) \mapsto \mathscr{E}(k\tau, u, z) + \mathscr{R}(z - z_\tau^{k-1}) \\ \text{subject to} & (u, z) \in \mathscr{U} \times \mathscr{Z},\end{array}\right\} \tag{8}$$

to be solved successively for $k = 1, ..., T/\tau$, starting from $u_\tau^0 = u_0$ and $z_\tau^0 = z_0$. By the standard direct method, existence of solutions to (8) is due to weak lower semicontinuity of $\mathscr{E}(t, \cdot, \cdot)$ and coerciveness of $\mathscr{E}(t, \cdot, \cdot) + \mathscr{R}(\cdot - z_\tau^{k-1})$. A solution in a time step k is then denoted as (u_τ^k, z_τ^k).

Comparing the energy value of a solution at the k^th-step with that of a solution $(u_\tau^{k-1}, z_\tau^{k-1})$ of the incremental problem (8) at the $(k-1)^{\text{th}}$-step we may derive an upper estimate of the energy balance in the k^{th}-step. Moreover, writing the stability condition (7) at the level $k-1$ and testing it by (u_τ^k, z_τ^k) gives a lower estimate of the energy balance in the k^{th}-step. Defining also $\overline{\mathscr{E}}_\tau(t, u, z) := \mathscr{E}(k\tau, u, z)$, $\overline{u}_\tau(t) = u_\tau^k$, $\overline{z}_\tau(t) = z_\tau^k$, $\underline{u}_\tau(t) = u_\tau^{k-1}$, and $\underline{z}_\tau(t) = z_\tau^{k-1}$ for $t \in ((k-1)\tau, k\tau]$, the following *two-sided energy estimate* may be established:

$$\begin{aligned}\int_0^s \mathscr{E}_t'\big(t, \overline{u}_\tau(t), \overline{z}_\tau(t)\big)\,\mathrm{d}t \\ \le \overline{\mathscr{E}}_\tau\big(s, \overline{u}_\tau(s), \overline{z}_\tau(s)\big) + \mathscr{R}(\overline{u}_\tau(s) - z_0) - \overline{\mathscr{E}}_\tau(0, u_0, z_0) \\ \le \int_0^s \mathscr{E}_t'\big(t, \underline{u}_\tau(t), \underline{z}_\tau(t)\big)\,\mathrm{d}t\end{aligned} \tag{9}$$

for any $s = k\tau$, $k = 1, ..., T/\tau$; here we used also that simply $\mathrm{Diss}_{\mathcal{R}}(\overline{z}_\tau, [0, s]) = \mathscr{R}(z_\tau^k - z_0)$.

This implicit time discretisation serves as a theoretical tool to prove extistence of the energetic solutions themselves. Moreover, the two-sided energy estimate (9) may facilitate the numerical solution of the global-optimization problem (8), as it can be used for a solution improvement by backtracking if the numerical minimization procedure did not work succesfully, as described, for example, in [13].

To implement the scheme computationally, one must make still some spatial discretisation. Here we use the boundary-element method (BEM). The BEM is closely related to the map between the prescribed boundary conditions in displacements or tractions and the unknown boundary displacements or tractions. In pure Dirichlet and Neumann BVPs these maps are called Steklov-Poincaré and Poincaré-Steklov maps, respectively, and BEM can be considered as an approach to discretize these maps.

In the present computational procedure, the role of the BEM analysis applied to each subdomain Ω_i separately (which in fact, makes this problem very suitable for parallel computers) is to solve the corresponding BPVs on Ω_i. Then, by using the computed values of boundary displacements and tractions the elastic strain energy stored in all Ω_i is evaluated in each time step and in each iteration of the minimization algorithm.

For this goal, we numerically solve the Somigliana displacement identity [9]

$$c_{kl}(\xi) u_k(\xi) + \fint_{\partial\Omega_i} u_k(x) T_{kl}(x, \xi)\,\mathrm{d}S = \int_{\partial\Omega_i} t_k(x) U_{kl}(x, \xi)\,\mathrm{d}S, \tag{10}$$

where $\xi \in \partial\Omega_i$, $u_k(x)$ and $t_k(x)$ denote the k^{th} component of displacement and traction vector, respectively. The weakly singular integral kernel $U_{kl}(x, \xi)$, two-point tensor field, given by the Kelvin fundamental solution (free-space Green's function) represents the displacement at x in the k-direction originated by a unit point force at ξ in the l-direction in the (unbounded) elastic medium in three dimensional space. The strongly singular integral kernel $T_{kl}(x, \xi)$, two-point tensor field, represents the corresponding tractions at x in the k-direction. The coefficient-tensor $c_{kl}(\xi)$ of the free-term is a function of the local geometry of the boundary $\partial\Omega_i$ at ξ, and may be evaluated by a closed analytical formula for isotropic elastic solids [18]. The symbol $⨍$ stands for the Cauchy principal value of an integral.

Consider a discretisation of the boundary $\partial\Omega_i$ by a boundary element mesh, which is also used to define a suitable discretisation of boundary displacements u_h and tractions t_h by interpolations of their nodal values. By imposing (collocating) the Somigliana identity (10) at all boundary nodes (called collocation points) we set the BEM linear system of equations. The solution of this system defines the unknown nodal values of u_h and t_h along $\partial\Omega_i$.

Then, to compute an approximation of the elastic energy stored in the bulk $\partial\Omega_i$ by using the obtained approximations of boundary displacements u_h and tractions t_h, at each time step, we utilize the following general relation, neglecting body forces but involving the boundary data $u_{\rm D}(t)$:

$$\mathscr{E}_{\Omega h}(t,\mathbf{u}_c) = \sum_{i=1}^{N} \mathscr{E}_{\Omega_i h}(t,\mathbf{u}_c) \quad \text{with} \quad \mathscr{E}_{\Omega_i h}(t,\mathbf{u}_c) = \frac{1}{2}\int_{\partial\Omega_i} t_h u_h \,\mathrm{d}S \tag{11}$$

with $\mathbf{u}_c$ and $\mathbf{z}$ being the vectors of nodal values of u_h and z_h defined on $\Gamma_{\rm C}$. The energy stored in the adhesive along the dissipative interfaces can be evaluated precisely by

$$\mathscr{E}_{\Gamma h}(\mathbf{u}_c,\mathbf{z}) = \int_{\Gamma_{\rm C}} z_h \Big(\frac{\kappa_{\rm n}}{2}\big|[\![u_h]\!]_{\rm n}\big|^2 + \frac{\kappa_{\rm t}}{2}\big|[\![u_h]\!]_{\rm t}\big|^2\Big)\,\mathrm{d}S, \tag{12}$$

where an approximation the damage variable z_h defined by using the same boundary element mesh as for displacements and tractions is considered. Similarly, the dissipated energy can be evaluated precisely by putting $\mathscr{R}_h(\mathbf{z}) := \mathscr{R}(z_h)$ with z_h determined by $\mathbf{z}$.

According to equations (11)-(12), the discretized minimization problem has now a boundary only form and it may be given in terms of the boundary nodal values as follows,

$$\left.\begin{array}{ll} \text{minimize} & (\mathbf{u}_c,\mathbf{z}) \mapsto \mathscr{E}_h(k\tau,\mathbf{u}_c,\mathbf{z}) + \mathscr{R}_h(\mathbf{z}-\mathbf{z}^{k-1}) \\ \text{subject to} & \mathbf{B}_I\mathbf{u}_c \ge \mathbf{0}, \quad \mathbf{0} \le \mathbf{z} \le \mathbf{z}^{k-1} \end{array}\right\} \tag{13}$$

where $\mathscr{E}_h(t,\mathbf{u}_c,\mathbf{z}) = \mathscr{E}_{\Omega h}(t,\mathbf{u}_c) + \mathscr{E}_{\Gamma h}(\mathbf{u}_c,\mathbf{z})$ while $\mathbf{B}_I$ represents the non-penetration Signorini conditions. Problem (13) is in general a difficult non-convex optimization problem that has to be numerical solved repeatedly at each time step. Several techniques have been used to solve efficiently this problem, such as the *alternating minimization algorithm* and a *heuristic back-tracking algorithm* which is based on the discrete form of the two-sided inequality (9). These techniques [13] have been modified appropriately for this work to fit in the current BEM implementation.

4 Numerical Results

The above introduced formulation has been implemented in a two-dimensional BEM code [19] using continuous piecewise linear boundary elements [9]. With reference to Figure 1, only one subdomain (i.e. $N = 1$) is used to model in a simple way an experimental test motivated by the pull-push shear test used in engineering practice [20]. The geometry of

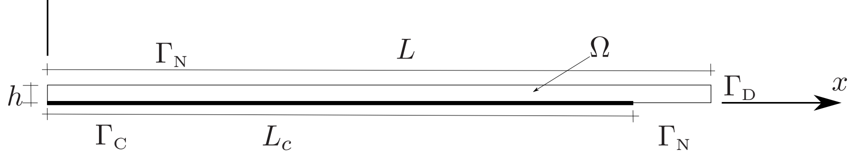


Figure 2: Geometry and boundary conditions of the problem considered.

the problem is shown in Figure 2. The length and height of the rectangular domain Ω, respectively, are $L = 250$ mm and $h = 12.5$ mm. The length of the initially glued part Γ_C placed at the bottom side of Ω is $L_c = 0.9L$. The elastic material of the bulk is an aluminium with the elasticity modulus $E = 70$ GPa and the Poisson's ratio $\nu = 0.35$. Elastic plain strain state is considered. The adhesive layer is represented of the normal stiffness κ_n =150 GPa/m, the tangential stiffness $\kappa_\mathrm{t} = \kappa_\mathrm{n}/2$ and the mode I fracture toughness $a_\mathrm{I} = 187.5$ J/m^2. The critical stress for the normal direction is defined by $\sigma_c = \sqrt{2\kappa_\mathrm{n} a_\mathrm{I}}$ =0.237 GPa. A hard device loading is assumed by prescribing horizontal and vertical displacements, respectively w_x and w_y=0, at the right-hand side of the rectangle $\partial\Omega$, defining the Dirichlet boundary Γ_D. All the other boundary parts, defining the Neumann boundary Γ_N, except for the contact surface Γ_C are considered to be traction free on $\partial\Omega$.The uniform boundary element mesh used to discretize the boundary $\partial\omega$ has 210 elements, 100 elements along each horizontal side and 5 elements per vertical side. Thus, Γ_C is discretized by 90 elements. The problem evolution is described in terms of a fictitious time t which is assumed to have unit value for a horizontal displacement equal to $\tilde{w}_x$ =0.1368mm.

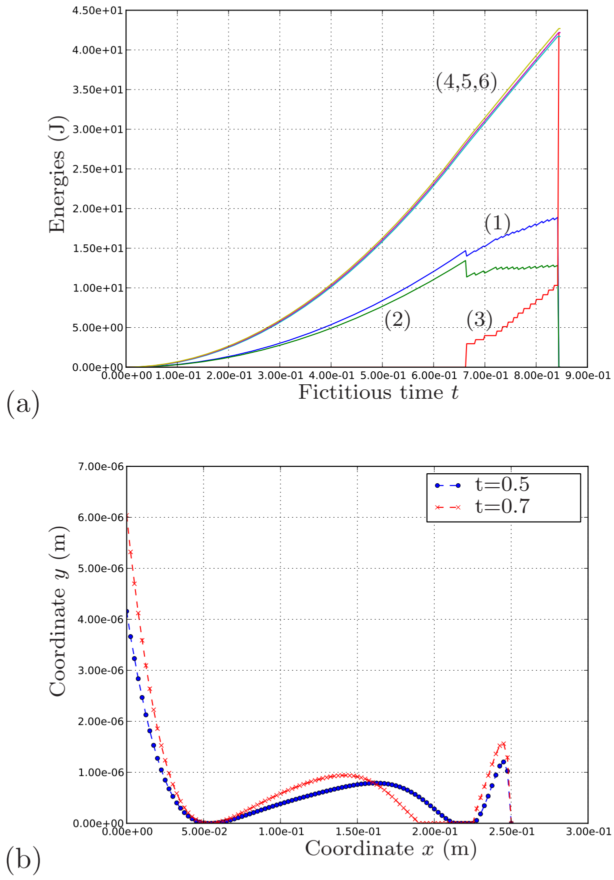
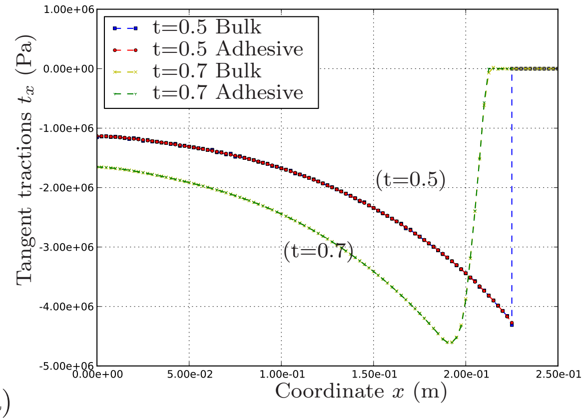
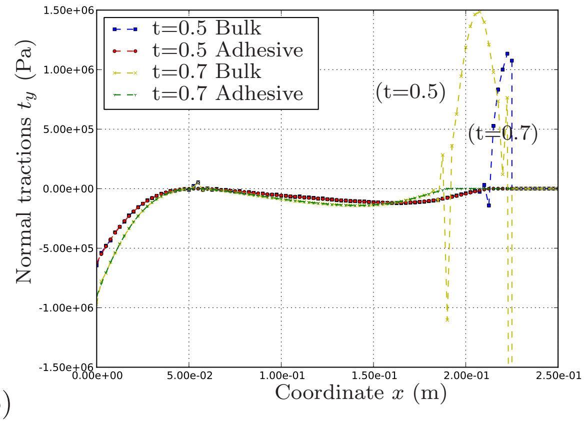


Figure 3: a) Energy evolution for (1) bulk, (2) surface, (3) dissipated, (4) lower-estimation, (5) total, (6) upper-estimation. b) Deformed shape of adhesive layer for two time steps, in the undamaged and damaged state

Figure 3a) shows the evolution of different energies with the fictitious time computed. In particular the energy stored in elastic bulk, additionally in adhesive layer and the dissipated energy. Also are shown the total energy, which is actually minimized in the time stepping procedure, together with the lower and upper estimates of energy (9). As it can be seen in Figure 3a), the global minimization procedure defines as the end point where the whole adhesive zone is debonded, the point where the sum of the energies in the bulk, the surface and the dissipated energy, is equal to energy needed for the total delamination of the whole adhesive zone. In Figure 3b), the deformed shape of the bottom of the elastic medium, or equivalently, the upper side of the adhesive, is plotted. Two contact zones could observed in these plots, where Signiorini contact conditions are valid. The resulted deformed shapes are given for two discrete time steps, one for an undamaged state (t=0.5) as well as one for a partially damage of state (t=0.7).



(a)

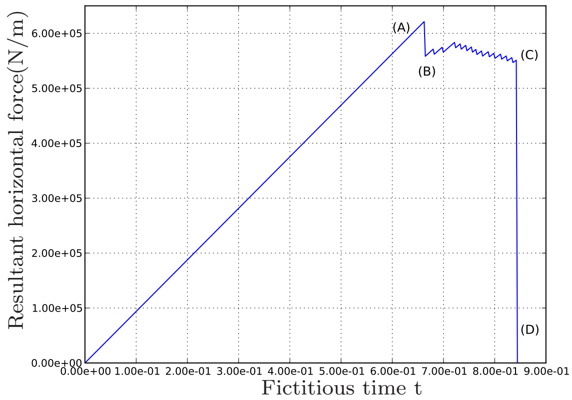


(b)

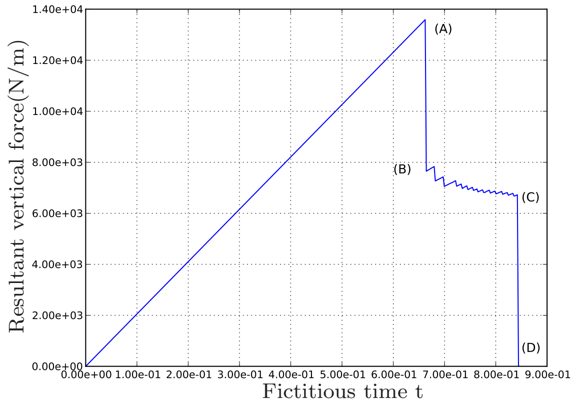
Figure 4: Tractions along the ashesive layer in a) tangent and b) normal direction computed by BEM for the bulk and those in the adhesive layer.

Figure 4, presents the components of the tractions vector along $\Gamma_{\rm C}$. As it can be observed there, very good agreement exists between the computed tractions in the elastic bulk by BEM implementation and that computed in adhesive layer, the equilibrium has not been included directly but results as a consequence of the energy minimization. Progressive extension of the traction free portion of the original $\Gamma_{\rm C}$ because of apperance of the total damage (z=0) can be observed in Figure 4a. It should be mentioned here that the portion, of the adhesive layer, which is totally damaged is still kept as a part of the minimization procedure where nodal displacement values participate as part of it set of unknowns in the minimization procedure and their values are used in the BEM solution of the pertinent boundary value problem. For this, we have a computed approximation of the developed free of traction zone. Obviously other algorithms might be used where after total damage a change to the type of the boundary condition might be taken into account in the BEM computation. Nevertheless we have been interested in the results obtained by this direct procedure. In the plots of Figure 4b, one may observe the compression zones, where zero values of tractions are obtained in the adhesive. This due to the fact that the rigid obstacle undertake these forces.

Finally, in Figure 5 there are plotted the resultant forces with displacements applied



(a)



(b)

Figure 5: Horizontal and vertical resultant force on the side where displacements are prescribed, evolution in fictitious time. With the letters (A)–(D) some characteristing points of the overall behaviour mentioned.

on Γ_D. These two plots have some similarities in the behaviour that may come up through the characteristic points (A)–(D). Up to point (A) the linear elastic behaviour manifests in both the solid and the adhesive, at this point the first damage appears, in the first 6 elements that are situated on the right side of the adhesive layer. This new crack length results in a "jump down" of the resultant forces up to point (B). Then, up to point (C) the damaged zone is progressively extended and finally after point (C) the remaining adhesive zone is then been damaged instantaneously ending at point (D), where rigid body motion of the elastic body takes place. We have to mention here that for the damage variable, as well as for the displacements and tractions, linear distribution over the elements has been considered, which results in some difficulty in the definition of the crack length. The case of constant distribution of damage variable z along each element will be presented in a forthcoming work [12].

5 Conclusions

In this work, the problem of the onset and propagation of an *elastic-brittle* or *purely brittle* delamination have been considered. A numerical procedured based on the framework of energetic solutions and numerically implemented by BEM was developed and presented. Numerical solution of an *elastic-brittle* delamination problem has been presented to demonstrate the capabilities of this new approach. The whole approach is under an ongoing research while under preperation is a forthcoming work where numerical results also for the *purely brittle* case will be incorporated as well as further inquiry on the possible consideration of seeking *local minima* and comparison with the pertinent global ones.

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