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IDENTIFICATION OF PREISACH-TYPE HYSTERESIS OPERATORS

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Abstract. Identification of a Preisach-type operator using a suitable fixed ridge functions is shown to be a well-posed problem and its discretization is proved to converge and is implemented by an adaptive manner.

Key words. Hysteresis, Krasnoselskiĭ-Pokrovskiĭ operators, identification.

AMS Subject Classification: 47H99, 65K10, 90C55, 93B30.

1 INTRODUCTION

The phenomenon of hysteresis is, by definition, just a rate-independent nonlinearity characterized by a memory effect. Hysteresis operators usually describe this phenomenon through the use of a single time-varying input and a single output. Motivated by hysteresis observed in magnetization transient response of ferromagnets, Preisach [14] introduced a basic model. This basic model was used to describe a large class of hysteretic responses having the so-called return-point property [3, 4]. During development of theory of hysteresis operators, the original Preisach operator was later modified and generalized in many ways, see [4, 9, 13, 16] for a thorough survey. A particular straightforward modification of Preisach's operator consists in replacing the Heaviside "ridge function" by a general non-decreasing (and usually continuous) ridge function r. The resulting operator is a special case of a so-called generalized play [7]. In engineering literature it has been referred as a Krasnoselskiĭ-Pokrovskiĭ (=KP) hysteresis operator. The definition of the KP operator requires, in addition to this ridge function r, also a positive measure μ on the Preisach plane of the threshold pairs (see Sect. 2 below for details). For a chosen ridge function the induced class of KP-operators is a subset of the class of all Preisach operators. This fact is made precise, for example, in [16]. In fact, if r is Lipschitz continuous, responses with derivatives bigger than the Lipschitz constant of r certainly cannot be obtained by the

KP-operator based on this r. Anyhow, the KP-operators are popular in engineering community because there are often well fitted in particular cases. Moreover, such operators facilitate various mathematical considerations better than general Preisach operators, as we will also see in this paper and was already observed in [1, Sect.4]. They may also be suited to specific control applications because of their invertibility in contrast to general Preisach operators.

In addition, hysteretic responses often arise from distributed-parameter physical systems. Replacing such usually complicated systems (or their mathematical/computational models) by suitable simple hysteresis operators can facilitate an efficient modeling of systems involving such hysteretic components (provided the working loading regime can be described as a single input). Beside mere modeling, it cannot be overemphasized that this may enable us to design efficient feedback control strategies in case when the hysteresis operator is (possibly at least approximately) invertible, cf. [12, 15, 17, 19].

Usually, the explicit form of the hysteresis operator can hardly be derived directly from the equations governing a distributed-parameter physical system in question. Thus, one has to identify the hysteresis operator in a pre-selected class by a data-fitting approach. Various methodologies have been developed in this context, cf. [1, 2, 5, 15]. The goal of this paper is to justify mathematically a particular methodology and to develop an adaptive algorithm whose efficiency is documented in a test example. In particular, Lemma 2.1 and Proposition 3.1 below extend the continuity and stability results in [1, 2].

Let us only remark that, in the context of control theory, invertibility of the KP-operators obtained from the identification process can advantageously be exploited. This is touched upon in [17, 18, 19] and also [10, 11], but we will not address this issue here.

2 PARAMETERIZED KP-OPERATORS

To be more specific, we use a parameterized version of an integral hysteresis operator incorporating the notion of Krasnoselskiĭ and Pokrovskiĭ's generalized play, see [7, 16]. The integral hysteresis operator is obtained by integration over a class of kernel functions, $\kappa_{(h_1,h_2)}$, each representing idealized elementary hysteresis operators [1]. The elementary operators are parameterized here by a couple (h_1, h_2) designating translates and assuming naturally $h_1 \leq h_2$, cf. (4)–(5) below. The integral hysteresis operator can then be written as

$$\big[P_\mu(v, m_0)\big](t) \equiv \iint_{\mathbb{P}} \big[\kappa_{(h_1,h_2)}(v, m_0(h_1, h_2)\big](t)\mu(\mathrm{d}h_1{\times}\mathrm{d}h_2) \tag{1}$$

where $\mathbb{P} := \{(h_1, h_2) \in \mathbb{R}^2;\ h_1 \leq h_2\}$ is the so-called *Preisach plane* (in fact, it is rather a half-plane), $m_0 : \mathbb{P} \to \mathbb{R}$ is a function of initial states, and $\mu \in \mathbf{M}(\mathbb{P})$=the set of all bounded Borel measures on $\mathbb{P}$. It is reasonable to consider $\mu \geq 0$ which will automatically ensure monotone-in-time response on monotone-in-time input provided r used below in (4) is nondecreasing. We will still consider the constraint on the total mass of μ, i.e. $\mu(\mathbb{P}) \leq M$ with $M < +\infty$ suitably large but fixed in order to ensure a-priori weak* compactness of the set of admissible measures, which is appropriate for optimization algorithms as well as for a convergence analysis. Sometimes even $\mu(\mathbb{P}) = 1$, i.e. μ is a probability measure, is selected in engineering literature. In general, this constraint is not a justified requirement and may lead to unnecessary misfit, however. Due to the continuous nature of $\kappa_{(h_1,h_2)}$, which was important for proving the well-posedness of identification strategies in [1, 2], we chose the KP operator to be developed further for adaptive identification.

Each of the elementary hysteresis operators then defines a mapping

$$\kappa_{(h_1,h_2)}(\cdot, m_0) : C[0,T] \to \mathcal{F}[0,T] \tag{2}$$

where $m_0 \in \mathbb{R}$ represents the initial state of the kernel and $\mathcal{F}[0,T]$ is a function space of outputs. For a fixed input $v \in C[0,T]$, the KP kernel has the following continuity properties:

$$t \mapsto \big[\kappa_{(h_1,h_2)}(v, m_0)\big](t) \quad \text{is continuous,} \tag{3a}$$

$$(h_1, h_2) \mapsto \big[\kappa_{(h_1,h_2)}(v, m_0)\big](t) \quad \text{is continuous,} \tag{3b}$$

cf. [1]. The time continuity of an operator is important for physical considerations, while the parametric continuity is useful for designing well-posed identification methodologies.

To be more specific, for the KP integral hysteresis operator, $\kappa_{(h_1,h_2)}$ is a form of generalized play, depicted in Figure 1(right). Specifying a nondecreasing continuous *ridge function* $r : \mathbb{R} \to \mathbb{R}$, the KP kernel $\kappa_{(h_1,h_2)}$ will first be defined for piecewise monotone functions, and then extended to all functions $v \in C[0,T]$. For any initial state m_0 and any monotone function v such that $r(v(0) - h_2) \leq m_0 \leq r(v(0) - h_1)$, we define the output operator by

$$\big[M(v, m_0)\big](t) = \begin{cases} \max\big\{m_0, r(v(t) - h_2)\big\} & \text{if } \frac{\mathrm{d}}{\mathrm{d}t}v(t) \geq 0, \\ \min\big\{m_0, r(v(t) - h_1)\big\} & \text{if } \frac{\mathrm{d}}{\mathrm{d}t}v(t) \leq 0. \end{cases} \tag{4}$$

Suppose that v is piecewise monotone, specifically a set of piecewise linear splines with j knots. The KP kernel is then defined by setting $M_0 = m_0$ and

$$\big[\kappa_{(h_1,h_2)}(v, m_0)\big](t) = \begin{cases} \big[M(v, M_{k-1})\big](t) \quad \text{for} \quad t \in [t_{k-1}, t_k] \\ \text{where } M_k = \big[M(v, M_{k-1})\big](t), \quad k = 1, \ldots, j. \end{cases} \tag{5}$$

The extension to all $v \in C[0,T]$ follows from continuity arguments discussed in [16].

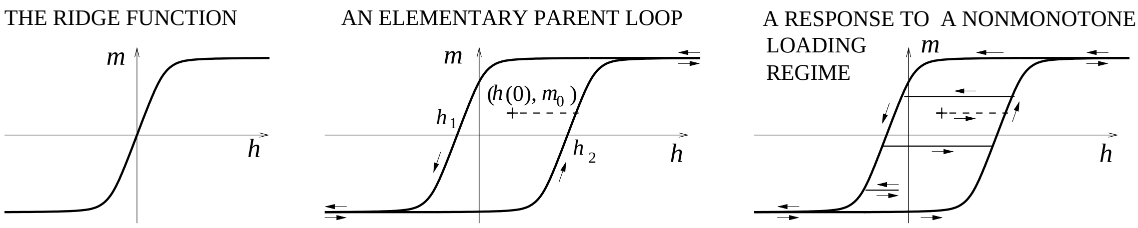

Fig. 1.

Left: *An illustration of a ridge function $r = r(h)$.*
Middle: *An elementary loop created by the pair of thresholds (h_1, h_2) and an example of the response due to (4) for an increasing load starting from $h(0)$ if the initial state is m_0.*
Right: *The response to a more complicated nonmonotone loading due to (5).*

Lemma 2.1 *Let $r \in W^{1,\infty}(\mathbb{R})$ monotone be given, K compact and let $\mathcal{M} = \{\mu \in \mathbf{M}(\mathbb{P})$, nonnegative, $\mathrm{supp}(\mu) \subset K\}$. Then the mapping*

$$(\mu, v, m_0) \mapsto P_\mu(v, m_0) : \mathcal{M} \times C([0,T]) \times C(\mathbb{P}) \to C([0,T])$$

is (weak×norm×norm,norm)-continuous.*

Remark 2.2 As $\mathrm{supp}(\mu_k) \in K$ is compact it is sufficient to have strong convergence $m_k \to m_0$ in $C(K)$. From the proof below we will see that the behavior of m_k outside K is not important.

Proof of Lemma 2.1. Take $v_k \to v$ strongly in $C([0,T])$, $m_{0,k} \to m_0$ strongly in $C(\mathbb{P})$ and $\mu_k \to \mu$ weakly* in $\mathbf{M}(\mathbb{P})$ and $\mathrm{supp}(\mu_k) \in K$ compact. We will prove that $P_{\mu_k}(v_k, m_{0,k}) \to P_\mu(v, m_0)$ strongly in $C([0,T])$.

Let us abbreviate the difference $D_k(t) := P_{\mu_k}(v_k, m_{0,k})(t) - P_\mu(v, m_0)(t)$. First let us show that D_k is bounded in $W^{1,\infty}(0,T) \Subset C([0,T])$ provided $r \in W^{1,\infty}(\mathbb{R})$. This is obvious from the definition, because for example

$$\sup_{t\in[0,T]} \left|P_{\mu_k}(v_k, m_{0,k})(t)\right| \le \int_{\mathbb{P}} \sup_{t\in[0,T]} \left|\kappa_{(l,s)}(v, m_0(l,s))(t)\right| \mathrm{d}\mu_k \le \|r\|_{L^\infty(\mathbb{R})} \int_{\mathbb{P}} \mathrm{d}\mu_k \le C,$$

$$\operatorname*{ess\,sup}_{t\in[0,T]} \left|\frac{\mathrm{d}}{\mathrm{d}t} P_{\mu_k}(v_k, m_{0,k})(t)\right| \le \int_{\mathbb{P}} \operatorname*{ess\,sup}_{t\in[0,T]} \left|\frac{\mathrm{d}}{\mathrm{d}t}\kappa_{(l,s)}(v, m_0(l,s))(t)\right| \mathrm{d}\mu_k$$

$$\le \|r'\|_{L^\infty(\mathbb{R})} \int_{\mathbb{P}} \mathrm{d}\mu_k \le C.$$

Note that $\kappa_{(l,s)}(...)$ is either constant or lies exactly on a shifted graph of r and therefore the derivative is either 0 or r', respectively. Now it is sufficient to show that $D_k \to 0$ almost everywhere in $[0,T]$. Take some $t \in [0,T]$ fixed. We want to show

$$\int_{\mathbb{P}} \kappa_{(l,s)}(v_k, m_{0,k}(l,s))(t) \mathrm{d}\mu_k \to \int_{\mathbb{P}} \kappa_{(l,s)}(v, m_0(l,s))(t) \mathrm{d}\mu. \tag{7}$$

It is known that $\kappa_{(.,.)}(v_k, m_{0,k}(.,.))(t) \in C(\mathbb{P})$ provided $m_0 \in C(\mathbb{P})$. Now we show that $\kappa_{(.,.)}(v_k, m_{0,k}(.,.))(t) \to \kappa_{(.,.)}(v, m_0(.,.))(t)$ strongly in $C(\mathbb{P})$.

Fix (l,s) then for all $m_1, m_2 \in \mathbb{R}$ and $v_1, v_2 \in C([0,T])$ the following inequality holds

$$\left|\kappa_{(l,s)}(v_1, m_1)(t) - \kappa_{(l,s)}(v_2, m_2)(t)\right| \le \max\left\{|m_1 - m_2|, \|r'\|_{L^\infty(\mathbb{R})}\|v_1 - v_2\|_{C([0,T])}\right\}. \tag{8}$$

Choose $\epsilon > 0$ arbitrarily. There exists a division of $[0,T]$ and piecewise monotone functions $\bar{v}_1$, $\bar{v}_2$ such that $\|\bar{v}_1 - v_1\|_{C([0,T])} < \epsilon$, $\|\bar{v}_2 - v_2\|_{C([0,T])} < \epsilon$ and $\mathrm{sgn}(\bar{v}_1') = \mathrm{sgn}(\bar{v}_2')$. From the definition (4) we have

$$\begin{aligned} |M(\bar{v}_1, m_1)(t) - M(\bar{v}_2, m_2)(t)| &\le \max\left\{|m_1 - m_2|, \left|r(\bar{v}_1(t) - c) - r(\bar{v}_2(t) - c)\right|\right\} \\ &\le \max\left\{|m_1 - m_2|, \|r'\|_{L^\infty(\mathbb{R})}\|\bar{v}_1 - \bar{v}_2\|_{C([0,T])}\right\} \end{aligned} \tag{9}$$

and, from the definition (5),

$$\begin{aligned} \left|\kappa_{(h_1,h_2)}(\bar{v}_1, m_1)(t) - \kappa_{(h_1,h_2)}(\bar{v}_2, m_2)(t)\right| &= \left|M(\bar{v}_1, M^1_{k-1})(t) - M(\bar{v}_2, M^2_{k-1})(t)\right| \\ &\le \max\Big\{\underbrace{|M^1_{k-1} - M^2_{k-1}|}_{\text{use again (9)}}, \|r'\|_{L^\infty(\mathbb{R})}\|\bar{v}_1 - \bar{v}_2\|_{C([0,T])}\Big\} \\ &\le \max\left\{\|M^1_0 - M^2_0\|, \|r'\|_{L^\infty(\mathbb{R})}\|\bar{v}_1 - \bar{v}_2\|_{C([0,T])}\right\} \end{aligned} \tag{10}$$

Here $M^i_k := [M(v_i, M^i_{k-1})](t)$ and $M^i_0 = m_i$, $i = 1, 2$. Hence (8) is proved for piecewise monotone functions with the same sign of the derivative. The rest follows by the continuity arguments.

Using (8) we have, for a.a. t,

$$\begin{aligned}\big|\kappa_{(l,s)}(v_k, m_{0,k}(l,s))(t) - \kappa_{(l,s)}(v, m_0(l,s))(t)\big| \\ \le \big\|m_{0,k} - m_0\big\|_{\mathbb{C}(\mathbb{P})} + \big\|r'\big\|_{L^\infty(\mathbb{R})}\big\|v_k - v\big\|_{C([0,T])}.\end{aligned} \tag{11}$$

The right-hand side is independent of (l,s) and converges to 0 as $k \to \infty$ so, in fact, we have $\kappa_{(\cdot,\cdot)}(v_k, m_{0,k}(\cdot,\cdot)) \to \kappa_{(\cdot,\cdot)}(v, m_0(\cdot,\cdot))$ strongly in $C(\mathbb{P})$. This, together with the weak* convergence of μ_k and $\text{supp}(\mu_k) \subset K$, ensures (7). As t was arbitrary, it holds $D_k \to 0$ a.e. in $[0,T]$. □

3 IDENTIFICATION OF THE MEASURE μ.

One computationally successful strategy, suggested already in [1, 2, 17, 18, 19], fixes the ridge function r for a given (fixed) initial state m_0 of the KP-operator as well as the input v. Subsequently, the measure μ is to be selected so that the response on that given loading regime v of the resulting KP-operator will be as close to each other (in a chosen norm) as possible. To be more specific, we consider the following optimization problem.

$$\left.\begin{array}{ll}\text{Minimize} & J(\mu) := \displaystyle\int_0^T |m(t) - m_{\mathrm{d}}(t)|^2 \mathrm{d}t \\ \text{subject to} & m := P_\mu(v, m_0) \ \text{ on } [0,T], \\ & \mu \ge 0, \ \ \text{supp}(\mu) \subset \mathbb{P}, \ \ \ \int_{\mathbb{P}} \mathrm{d}\mu \le M.\end{array}\right\} \tag{12}$$

The constant $M > 0$ is usually taken so large that the last constraint will not be active, and it facilitates and simplifies the theoretical analysis. Although Lemma 2.1 would suggest the use of a general L^p-norm in the cost functional in (12), it is advantageous to consider just the L^2-norm. This choice yields a convex linear/quadratic optimization problem because the KP-operator is obviously linear in terms of μ, cf. (1).

Proposition 3.1 ***(Stability of (12) under data perturbations.)*** *Let a nondecreasing $r \in W^{1,\infty}([0,T])$ be given. Then, for any $v \in C([0,T])$, $m_{\mathrm{d}} \in L^2(0,T)$, and $m_0 \in C(\mathbb{P})$, the optimization problem (12) has a solution μ. Moreover (12) is stable under perturbation of v and m_{d} and m_0 in the sense that the set-valued mapping $(v, m_{\mathrm{d}}, m_0) \mapsto S(v, m_{\mathrm{d}}, m_0) : C([0,T]) \times L^2(0,T) \times C(\mathbb{P}) \rightrightarrows \mathbf{M}(\mathbb{P})$, where $S(v, m_{\mathrm{d}}, m_0)$ denotes the set of all solutions to (12), is (norm,weak*)-upper semicontinuous.*

Proof. Nonemptyness of $S(v, m_{\mathrm{d}})$ follows by the usual compactness argument: the problem (12) has a weakly* compact set of admissible μ's on which the cost functional is weakly* continuous because of Lemma 2.1.

As for the stability of S, let us take a sequence $v_k \to v$ converging strongly in $C([0,T])$ and $m_{\mathrm{d},k} \to m_{\mathrm{d}}$ strongly in $L^2(0,T)$ and $m_{0,k} \to m_0$ strongly in $C(\mathbb{P})$. Due to the constraints in (12), $S(v_k, m_{\mathrm{d},k}, m_{0,k})$ are bounded in $\mathbb{M}(\mathbb{P})$ independently of k. Further, we consider some $\mu_k \in S(v_k, m_{\mathrm{d},k}, m_{0,k})$, which means that

$$\int_0^T \big|P_{\mu_k}(v_k, m_{0,k})(t) - m_{\mathrm{d},k}(t)\big|^2 \mathrm{d}t \le \int_0^T \big|P_{\tilde{\mu}}(v_k, m_{0,k})(t) - m_{\mathrm{d},k}(t)\big|^2 \mathrm{d}t, \tag{13}$$

holds for all $\tilde{\mu}$ admissible for (12). By the mentioned boundedness of $S(v_k, m_{\mathrm{d},k})$, up to a subsequence we have $\mu_k \to \mu$ weakly* in $\mathbb{M}(\mathbb{P})$. Using our assumptions and Lemma 2.1, we can pass to the limit on both sides of (13):

$$\begin{aligned}\int_0^T \left|P_\mu(v,m_0)(t)-m_{\mathrm{d}}(t)\right|^2 \mathrm{d}t \;&=\; \lim_{k\to\infty}\int_0^T \left|P_{\mu_k}(v_k,m_{0,k})(t)-m_{\mathrm{d},k}(t)\right|^2 \mathrm{d}t \\ &\le\; \lim_{k\to\infty}\int_0^T \left|P_{\tilde{\mu}}(v_k,m_{0,k})(t)-m_{\mathrm{d},k}(t)\right|^2 \mathrm{d}t \\ &=\; \int_0^T \left|P_{\tilde{\mu}}(v,m_0)(t)-m_{\mathrm{d}}(t)\right|^2 \mathrm{d}t \qquad (14)\end{aligned}$$

for all $\tilde{\mu}$ admissible for (12), which just means that $\mu \in S(v, m_{\mathrm{d}})$. □

However, the optimization problem (12) is still infinite-dimensional and to implement it on computers, we must still approximate μ. The option usually applied (see e.g. [2, 5, 12]) is to consider a subspace of $\mathbf{M}(\mathbb{P})$ of measures supported only on a fixed finite subset of $\mathbb{P}$. Let us denote such a subset by $\mathbb{P}_k$, say $\mathbb{P}_k = \{(h_{1l}, h_{2l})\in\mathbb{P};\ l = 1, ..., k\}$. The original measure μ is thus approximated by

$$\mu(\mathbb{P}_k, w_1, ..., w_k) := \sum_{l=1}^{k} w_l \delta_{(h_{1l},h_{2l})}, \quad \sum_{l=1}^{k} w_l \le M, \quad \forall l = 1, ..., k:\ w_l \ge 0. \qquad (15)$$

This form has the definite advantage that the resulted problem is finite-dimensional, preserves the linear/quadratic structure of the optimization problem, and the approximate KP-operator can be easily evaluated because the integral in (1) turns into a sum. Altogether, considering $T > 0$ a fixed time horizon and denoting by $m_{\mathrm{d}} \in C(0,T)$ the desired response on a given loading regime $v \in C(0,T)$ obtained from a model below, we arrive at the following linear/quadratic programming problem on the vector of weights $(w_1, ..., w_k)$:

$$\left.\begin{array}{ll}\text{Minimize} & J(\mu(\mathbb{P}_k, w_1, ..., w_k)) := \int_0^T |m(t)-m_{\mathrm{d}}(t)|^2 \mathrm{d}t \\ \text{subject to} & m := P_{\mu(\mathbb{P}_k,w_1,...,w_k)}(v,m_0) = \sum_{l=1}^{k} w_l \kappa_{(h_{1l},h_{2l})}(v,m_0) \ \text{ on } [0,T], \\ & \sum_{l=1}^{k} w_l \le M, \quad w_l \ge 0 \quad \text{for all } l = 1, ..., k.\end{array}\right\} \qquad (16)$$

Of course, the integral in (16) is to be calculated by a numerical integration but we will omit these details here.

Proposition 3.2 ***(Convergence of the discretizations.)*** *Let $r \in W^{1,\infty}([0,T])$ be nondecreasing, and $v \in C([0,T])$ and $m_0 \in C([0,T])$ be given. Suppose the discretizations $\mathbb{P}_k$ refine everywhere on the triangle $\triangle := \{(h_1,h_2);\ \min v([0,T]) \le h_1 \le h_2 \le \max v([0,T])\} \subset \mathbb{P}$ in the sense*

$$\lim_{k\to\infty} \max_{(h_1,h_2)\in\triangle} \min_{l=1,..,k} |h_{1l}-h_1| + |h_{2l}-h_2| = 0. \qquad (17)$$

Then the sequence $\{\mu(\mathbb{P}_k, w_1, ..., w_k)\}_{k\in\mathbb{N}}$ where $\mu(\mathbb{P}_k, w_1, ..., w_k)$ is some solution to (16) contains a subsequence converging weakly in $\mathbb{M}(\mathbb{P})$; let μ denote its limit. Moreover, every μ obtained by this way solves (12).*

Proof. For $v \in C([0,T])$ we can confine ourselves only to such μ's for which $\mathrm{supp}(\mu)$ is contained in the triangle $\triangle$ without increasing the minimal value of (12). The condition (17) ensures that any positive measure $\tilde{\mu} \in \mathbb{M}(\triangle)$ with $|\tilde{\mu}| \le M$ can be approximated weakly* by a sequence $\{\tilde{\mu}_k\}_{k\in\mathbb{N}}$ of the form $\tilde{\mu}_k = \sum_{l=1}^{k} \tilde{w}_l \delta_{(h_{1l},h_{2l})}$ with $\tilde{w}_l \ge 0$ such that $\sum_{l=1}^{k} \tilde{w}_l \le M$. We have

$$\int_0^T \Big|\big[P_{\mu(\mathbb{P}_k,w_1,...,w_k)}(v,m_0)\big](t) - m_{\mathrm{d}}(t)\Big|^2 \mathrm{d}t \le \int_0^T \Big|\big[P_{\tilde{\mu}_k}(v,m_0)\big](t) - m_{\mathrm{d}}(t)\Big|^2 \mathrm{d}t. \tag{18}$$

We can consider a subsequence such that $\mu(\mathbb{P}_k, w_1, ..., w_k) \to \mu$ weakly* in $\mathbb{M}(\triangle)$. Then, again by Lemma 2.1, we can pass to the limit on both sides of (18) which gives

$$\int_0^T \Big|\big[P_{\mu}(v,m_0)\big](t) - m_{\mathrm{d}}(t)\Big|^2 \mathrm{d}t \le \int_0^T \Big|\big[P_{\tilde{\mu}}(v,m_0)\big](t) - m_{\mathrm{d}}(t)\Big|^2 \mathrm{d}t. \tag{19}$$

Since inequality (19) holds for an arbitrary positive $\tilde{\mu} \in \mathbb{M}(\triangle)$ (in fact even for $\tilde{\mu} \in \mathbb{M}(\mathbb{P})$) with $|\tilde{\mu}| \le M$, it just proves that μ solves (12). □

4 IMPLEMENTATION, AN ADAPTIVE DISCRETIZATION OF PREISACH'S PLANE

The support of the measure in (15), i.e. the set $\mathbb{P}_k := \{(h_{1l}, h_{2l}) \in \mathbb{P};\ l = 1, \dots, k\}$, is not a-priori determined and has to be chosen somehow. A simple way is to approximate (a selected part of) the Preisach plane by a sufficiently and uniformly dense set of points, say $\mathbb{P}_k^{(0)}$, and then solve (16) to get the measure $\mu(\mathbb{P}_k^{(0)}, w_1^0, ..., w_k^0)$, see e.g. [2]. In applications the resulting measure is typically supported on a rather small area. Hence, one is certainly tempted to improve the original uniform Preisach-plane discretization $\mathbb{P}_k^{(0)}$ by exploiting the information about the support of $\mu(\mathbb{P}_k^{(0)}, w_1^0, ..., w_k^0)$ in an adaptive manner. This way we get a discretization $\mathbb{P}_k^{(1)}$ refined on the support of $\mu^{(0)}$ while coarsened elsewhere. Of course, we can follow this heuristic strategy recursively. To achieve this goal, we developed a routine which works with the Preisach plane as with a tree of triangles. The root is a triangle big enough to contain the whole support of the measure μ, e.g. $\triangle$ from Proposition 3.2. Each triangle can be divided into four smaller triangles. In the leaves are the smallest triangles which contain points of $\mathbb{P}_k$. The simple algorithm is as follows. If the density on the leaf triangle is greater than the prescribed threshold then split this triangle into four triangles and, conversely, if the mass on some subtree is less than another prescribed threshold then reduce this subtree to a single leaf. Great advantage is that we can evaluate the functional J for both $\mu(\mathbb{P}_k^{(\ell)}, w_1^\ell, ..., w_k^\ell)$ and $\mu(\mathbb{P}_k^{(\ell+1)}, w_1^{\ell+1}, ..., w_k^{\ell+1})$ to find out if the adaptive modification $\mathbb{P}_k^{(\ell+1)}$ of $\mathbb{P}_k^{(\ell)}$ was successful, i.e. if $J(\mu(\mathbb{P}_k^{(\ell+1)}, w_1^{\ell+1}, ..., w_k^{\ell+1})) \le J(\mu(\mathbb{P}_k^{(\ell)}, w_1^\ell, ..., w_k^\ell))$. This checks a-posteriori whether the particular step of this adaptive algorithm has worked successfully.

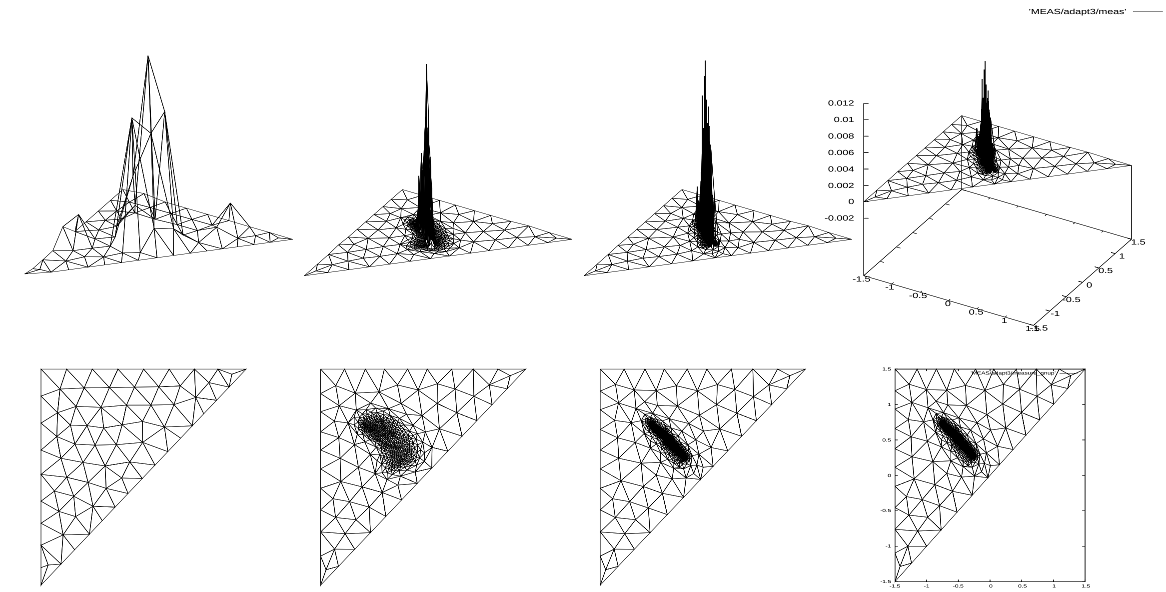

Fig. 2. Measure $\mu(\mathbb{P}_k^{(\ell)}, w_1^\ell, ..., w_k^\ell)$ and the supporting (fictitious) triangulation $\mathbb{P}_k^{(\ell)}$ during adaptive refinement/coarsening procedure iterations $\ell = 1$, 2, 4 and 7.

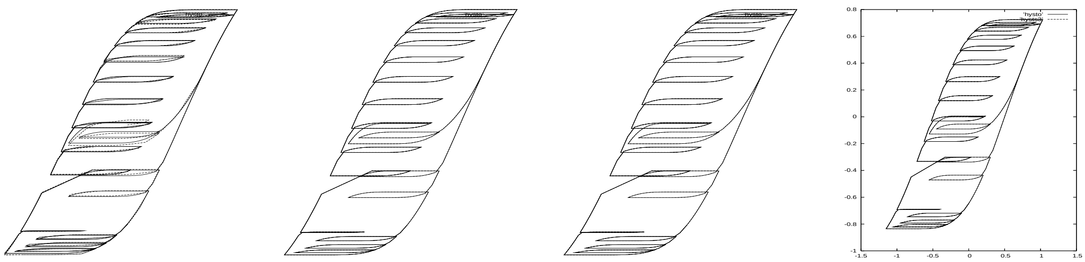

Fig. 3. The desired hysteretic response given by $m_{\rm d}$ and the responses obtained by subsequent iterations for $\ell = 1, 2, 4, 7$ from Figure 2.

We can alternate between refining the mesh uniformly everywhere (e.g. by dividing each triangle into four similar ones) and by the adaptive refinement/coarsening procedure. The overall convergence will still be preserved if (17) is kept holding. In reality, however, the adaptive procedure speeds up convergence while keeping the number of variables relatively small.

Remark 4.1 *(Nonuniqueness.)* One can easily construct an example wherein the LQ-program (12) need not be positive definite at least if the "training" function $m_{\rm d}$ is not sufficiently oscillatory. Therefore, nonuniqueness of the solution μ can occur. That is, $S(v, m_{\rm d}, m_0)$ in Proposition 3.1 need not be a singleton. This is depicted schematically in Figure 4, assuming that the training function $m_{\rm d}$ is just a simple cycling that ranges only over the parent loop.

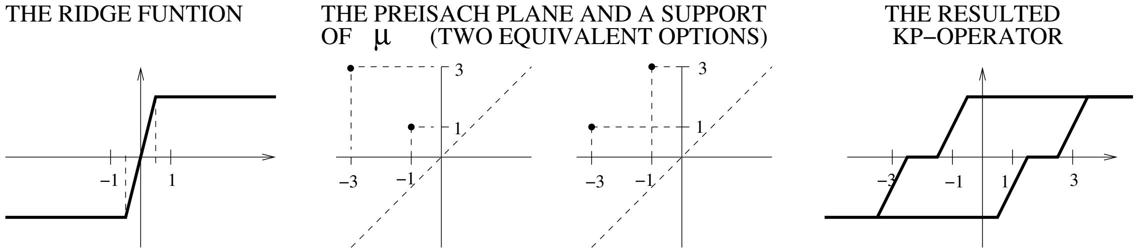

Fig. 4. Example on non-uniqueness: both $\mu = \frac{1}{2}\delta_{(-h_1,h_1)} + \frac{1}{2}\delta_{(-h_2,h_2)}$ *and* $\mu = \frac{1}{2}\delta_{(-h_2,h_1)} + \frac{1}{2}\delta_{(-h_1,h_2)}$ *are solutions if only the parent loop is traced.*

A reasonable goal is to get rid off this artificial nonuniqueness which may smear out any information of our interest. In view of Figure 4, an obvious strategy, pioneered already in [5], is to consider a sufficiently rich oscillatory character of the input $m_{\rm d}$. Another strategy would be to consider additional constraints on $\mathbb{P}_k$. For example, if the system producing hysteretic response $m_{\rm d}$ is presumably symmetric under changing (m, h) for $(-m, -h)$, then it makes sense to seek only measures supported near the side-diagonal $\{h_1 = -h_2\} \subset \mathbb{P}$. Then one can treat it, e.g., by a Tikhonov regularization. Presumably, chances for uniqueness of the identified measure increases this way.

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