NAFSA 9 — SHORT TALKS

1. Ali Akbulut (Ahi Evran University, Kirsehir, Turkey), *Boundedness of the maximal operator and singular integral operator in generalized Morrey spaces.* In this talk we consider the conditions on the pair (ω_1, ω_2) which ensures the boundedness of the maximal operator and Calderón-Zygmund singular integral operators from one generalized Morrey space $\mathcal{M}_{p,\omega_1}$ to another $\mathcal{M}_{p,\omega_2}$, $1 < p < \infty$, and from the space $\mathcal{M}_{1,\omega_1}$ to the weak space $W\mathcal{M}_{1,\omega_2}$. As applications, by these results we get some estimates for uniformly elliptic operators on generaized Morrey spaces.

(Joint work with V. Guliyev, R. Mustafaev.)

2. Jürgen Appell (University of Würzburg, Würzburg, Germany), *Degeneracy Phenomena for Composition Operators.* Given a Banach space X of functions $f : [a, b] \to \mathbb{R}$ and a fixed function $g : \mathbb{R} \to \mathbb{R}$, we consider the (autonomous) nonlinear composition operator T_g defined on X by $T_g(f) = g \circ f$. In particular, we will be interested in the following questions:

- Which conditions on g, possibly both necessary and sufficient, ensure that T_g maps X into itself?
- Which conditions on g, possibly both necessary and sufficient, ensure that T_g satisfies a global Lipschitz condition in the norm of X?
- Which conditions on g, possibly both necessary and sufficient, ensure that T_g satisfies a local Lipschitz condition in the norm of X?

The purpose of this talk is to show that the answer to these harmless looking questions exhibits many unexpected features for several important function spaces X. It turns out that, loosely speaking, the answer to the first and third question looks rather the same for a large variety of spaces X, while the answer to the second question leads to a strong degeneracy phenomenon for the the underlying function g, but only in some special spaces X. Both the results and techniques discussed in the talk are elementary and do not require a profound background in nonlinear analysis.

3. Mehdi Asadi (Islamic Azad University, Zanjan Branch, Zanjan, Iran), *Contractions in the cone metric spaces can be obtained via ordinary metric.* The main aim of the lecture is to investigate the equivalence of fixed point theorems in cone metrics spaces and scalar version of fixed point theorems in ordinary metric spaces. We prove that every cone metric space is metrizable and the equivalent metric satisfies the same contractive conditions as the cone metric. By our results most of the well-known fixed point theorems in cone metric spaces are straightforward results from the ordinary metric spaces.

(Joint work with S.M. Vaezpour.)

4. Radek Cibulka (University of Western Bohemia, Plzeň, Czech Republic), *Exact and approximate controllability of dynamic systems.* We consider an infinite-dimensional dynamic system described by a semi-linear abstract differential equation. The controls are subjected to constraints given by a closed convex subset of $L^\infty([0,T], U)$, where U is a real Banach space and $T > 0$. Sufficient conditions for both the approximate and exact local controllability in time T are proved via a constrained analogue of the well-known Graves' theorem.

5. Robert Černý (Charles University, Praha, Czech Republic), *Generalized Moser-Trudinger inequality and its applications.* Let Ω be bounded domain in $\mathbb{R}^n$, $n \geq 2$, and let $\alpha < n-1$. We prove the Concentration-Compactness Principle of P.L. Lions for the embedding of the Orlicz-Sobolev space $W_0^1 L^n \log^\alpha L(\Omega) := W_0^1 L^\Phi(\Omega)$ (with a Young function $\Phi(t) \approx t^n \log^\alpha(t)$) into the Orlicz space $L^\Psi(\Omega)$ with the Young function $\Psi(t) = \exp(t^{\frac{n}{n-1-\alpha}}) - 1$.

Updated on September 26, 2010.

Applying the Mountain Pass Theorem we prove the existence of a non-trivial weak solution $u \in W_0^1 L^n \log^\alpha L(\Omega)$ to the Dirichlet problem

$$-\operatorname{div}\Big(\Phi'(|\nabla u|)\frac{\nabla u}{|\nabla u|}\Big) = f(x,u) \quad \text{in } \Omega\ ,$$

where $|f(x,t)| \approx \exp(\beta |t|^{\frac{n}{n-1-\alpha}})$, $\beta > 0$.

(Joint work with P. Gurka and S. Hencl.)

6. **Vagif S. Guliyev** (Ahi Evran University, Kirsehir, Turkey), *Boundedness of the maximal, potential and singular operators in the generalized variable exponent Morrey spaces.* We consider generalized Morrey spaces $\mathcal{M}^{p(\cdot),\omega}(\Omega)$ with variable exponent $p(x)$ and a general function $\omega(x,r)$ defining the Morrey-type norm. In case of bounded sets $\Omega \subset \mathbb{R}^n$ we prove the boundedness of the Hardy-Littlewood maximal operator and Calderon-Zygmund singular operators with standard kernel, in such spaces. We also prove a Sobolev-Adams type $\mathcal{M}^{p(\cdot),\omega}(\Omega) \to \mathcal{M}^{q(\cdot),\omega}(\Omega)$-theorem for the potential operators $I^{\alpha(\cdot)}$, also of variable order. The conditions for the boundedness are given it terms of Zygmund-type integral inequalities on $\omega(x,r)$, which do not assume any assumption on monotonicity of $\omega(x,r)$ in r.

(Joint work with J.J. Hasanov, S.G. Samko.)

7. **Maryia Kabanava** (Friedrich Schiller University of Jena, Jena, Germany), *Anisotropic spaces as isotropic spaces on fractals.* There are two ways of defining Besov spaces on a d-set Γ. We get $B^s_{pp}(\Gamma,\mu)$ by traces or we define $B^{s,\alpha}_{pp}(\Gamma,\mu)$ to be the image of anisotropic Besov spaces on the cube Q under some bi-Lipschitz transfrom. In the talk we answer the question how these spaces are interrelated for a certain range of parameters s and p.

8. **Petr Kaplický** (Charles University, Praha, Czech Republic), *L^q estimates for generalized Stokes problem.* We consider a weak solution u to generalized stationary Stokes problem, where the Laplace operator is replaced by a more general nonlinear operator $\operatorname{div} T$. The mapping T is assumed to depend only on symmetric gradient Du of the solution and to behave as $(1+|Du|)^{p-2}Du$ for some $p \in (1,+\infty)$. The natural regularity of the weak solution is $u \in W^{1,p}_{loc}$. We show that integrability of ∇u can be improved if the data of the problem are sufficiently good. We also discuss optimality of the results.

9. **Turhan Karaman** (Ahi Evran University, Department of Mathematics, Kirsehir, Turkey), *Weighted fractional Sobolev-Morrey spaces and pseudo-differential operators with smooth symbols.* We study the boundedness of 0-order pseudo-differential operators with smooth symbols on weighted Morrey spaces $L_{p,\kappa}(w)$ whenever the weight function belongs to Muckenhoupt's class A_p. We introduce the weighted fractional Sobolev-Morrey spaces $L^s_{p,\kappa}(w)$. We give two embedding theorems for the weighted fractional Sobolev-Morrey spaces and characterize the weighted Sobolev-Morrey spaces $W^k_{p,\kappa}(w)$, $k \in \mathbb{N}$. We show that if A are pseudo-differential operators with smooth symbols of order m , then A are bounded from the weighted fractional Sobolev-Morrey spaces $L^s_{p,\kappa}(w)$ to $L^{s-m}_{p,\kappa}(w)$ for $0 \le m \le s$.

(Joint work with V. Guliyev and A. Serbetci.)

10. **Henning Kempka** (Friedrich Schiller University of Jena, Jena, Germany), *Wavelet characterizations of anisotropic function spaces.* We want to present different approaches to give wavelet characterizations for anisotropic Besov and Triebel-Lizorkin spaces. We give an overview over wavelet decomposition theorems on these spaces by Bownik, Kyriazis and an own approach. We are interested in giving as sharp as possible versions of these theorems with respect to the needed smoothness and moment conditions for Daubechies wavelets.

11. **Martin Lind** (Karlstad University, Karlstad, Sweden), *On functions of bounded p-variation.* We obtain estimates of the total p-variation $(1 < p < \infty)$ and other related functionals for a periodic function $f \in L^p[0,1]$ in terms of its L^p-modulus of continuity. These estimates are sharp for any rate of the decay of $\omega(f;\delta)_p$. Moreover, the constant coefficients in them depend on parameters in an optimal way.

12. **Shinya Moritoh** (Nara Women's University, Nara, Japan), *An integral representation formula for potentials, embeddings of Bessel-potential spaces, and Lorentz-Karamata spaces.* It was proved in Moritoh-Tanaka (2009) that the embedding

$$H^{\sigma;\alpha_1,\alpha_2}L^{n/\sigma,p}(\log L)^{1/p'-\alpha_1}(\log\log L)^{\beta}(\mathbb{R}^n) \hookrightarrow L_{\Phi_{A,-\alpha}}(\Omega)$$

is sharp in the sense of Hempel-Morris-Trudinger (1970), where the source spaces have logarithmic exponents both in the smoothness and in the underlying Lorentz-Zygmund spaces, and $\alpha := \alpha_2 + \beta - 1/p' < 0$.

The aims of the talk are to give the embeddings of the above type with slowly varying functions and to obtain the estimates for the norms of the embeddings in $L^q(\Omega)$ in terms of the slowly varying functions.

A family of functions $\{f_r\}$, satisfying $\|f_r \mid L^{n/\sigma,p;b_2}\| \leq 1$ and that $|(g_{\sigma,b_1} * f_r)(x)|$ is large for small values of $|x|$, is constructed according to Edmunds-Gurka-Opic (2000) and Gogatishvilli-Neves-Opic (2004). Some properties of the generalised Bessel kernels g_{σ,b_1} are also described according to Opic-Trebels (2000).

It was proved in Edmunds-Gurka-Opic (1998) that the inequality

$$(2/\nu - \log\sigma)^{-1/\nu}s^{\sigma} \lesssim \log^{-1/\nu}(e - \log s)$$

for $0 < \sigma < 1$ and $0 < s < 1$ holds, where ν is a positive constant. The counterpart of the inequality with slowly varying functions is considered; for the functions f satisfying $\int_0^1 f(t)\,dt/t = \infty$, the inequality of the following type

$$\Big(\int_{a(\sigma)}^1 f(t)\,dt/t\Big)^{-1}s^{\sigma} \lesssim \Big(\int_s^1 f(t)\,dt/t\Big)^{-1}$$

is considered, where the function $a(\sigma)$ was explicit for logarithmic functions f in [EGO 98], e.g. $a(\sigma) = \exp(-1/\sigma)$.

(Joint work with Y. Tanaka.) A part of the talk was presented at the meeting of the Mathematical Society of Japan in March, 2010. We aim at some extensions of the results obtained by the above authors.

13. **Paolo Musolino** (University of Padova, Italy), *A functional analytic approach for a singularly perturbed Dirichlet problem for the Laplace operator in a periodically perforated domain.* This talk is devoted to the analysis of a singularly perturbed Dirichlet problem for the Laplace operator in a periodically perforated domain. We consider a sufficiently regular bounded open connected subset Ω of $\mathbb{R}^n$ such that $0 \in \Omega$ and such that the complement of the closure of Ω is connected. Then we choose a point $w \in]0,1[^n$. If ϵ is a small positive real number, then we consider the periodically perforated domain $T(\epsilon)$ obtained by removing from $\mathbb{R}^n$ the closure of the set $\cup_{z\in\mathbb{Z}^n}(w+\epsilon\Omega+z)$. For each positive ϵ we consider a particular Dirichlet problem for the Laplace operator in the set $T(\epsilon)$. More precisely, we consider a Dirichlet condition on the boundary of the set $w + \epsilon\Omega$, and we denote the unique periodic solution of this problem by $u[\epsilon]$. Then we study the behaviour of $u[\epsilon]$ as ϵ goes to 0, by an approach that aims at representing the solutions by means of real analytic functions of ϵ in a neighbourhood of 0. Such an approach, based on Functional Analysis and Potential Theory, is alternative to those of Asymptotic Analysis and of Homogenization Theory.
Based on joint work with Massimo Lanza de Cristoforis, University of Padova.

14. **Ayhan Serbetci** (Ankara University, Ankara, Turkey), *The Stein-Weiss type inequalities for the B-Riesz potentials.* We establish two inequalities of Stein-Weiss type for the Riesz potential operator $I_{\alpha,\gamma}$ ($B-$Riesz potential operator) generated by the Laplace-Bessel differential operator Δ_B in the weighted Lebesgue spaces $L_{p,|x|^\beta,\gamma}$. We obtain necessary and sufficient conditions on the parameters for the boundedness of $I_{\alpha,\gamma}$ from the spaces $L_{p,|x|^\beta,\gamma}$ to $L_{q,|x|^{-\lambda},\gamma}$, and from the spaces $L_{1,|x|^\beta,\gamma}$ to the weak spaces $WL_{q,|x|^{-\lambda},\gamma}$. In the limiting case $p = Q/\alpha$ we prove that the modified $B-$Riesz potential operator $\widetilde{I}_{\alpha,\gamma}$ is bounded from the space $L_{p,|x|^\beta,\gamma}$ to the weighted $B-BMO$ space $BMO_{|x|^{-\lambda},\gamma}$. As applications, we get the boundedness of $I_{\alpha,\gamma}$ from the weighted B-Besov spaces $B^s_{p\theta,|x|^\beta,\gamma}$ to the spaces $B^s_{q\theta,|x|^{-\lambda},\gamma}$. Furthermore, we prove two Sobolev embedding theorems on weighted Lebesgue $L_{p,|x|^\beta,\gamma}$ and weighted B-Besov spaces $B^s_{p\theta,|x|^\beta,\gamma}$ by using the fundamental solution of the B-elliptic equation $\Delta_B^{\alpha/2}$.

(Joint work with A.D. Gadjiev, V.S. Guliyev and E.V. Guliyev, to be published in *Journal of Mathematical Inequalities.*)

15. **Hossein Soleimani** (Islamic Azad University, Malayer, Iran), *Some stability results in fixed point theory for metric trees.* Kirk introduced the fixed point property for continuous mappings in complete metric trees. In this lecture, inspired of the kirk proof, we consider a complete metric space of all the continuous mappings on compact and convex subsets of the complete metric tree and we show that a typical element of this space has a fixed point which is stable under small perturbations of the mapping. We also prove some theorems in this spaces.

16. **Filip Strobin** (Institute of Mathematics, Polish Academy of Sciences, Warszawa and Institute of Mathematics, Technical University of Łódź, Łódź, Poland), *Some porous sets of continuous nonexpansive-type mappings.* The first purpose of our talk is to present a general result which states that the complement of the set of all generalized contractive mappings forms a σ-porous (in particular, meager) subset of the space of all generalized nonexpansive mappings.
The second purpose is to present a few applications of this result. Each of them states that a set of all mappings which have a certain fixed point property (for example, the existence of fixed point, satisfying the thesis of Banach Fixed Point Theorem, the existence of fixed point of multivalued mapping) is large in a "ider" space (e.g. the space of all nonexpanxive mappings, the space of all α-nonexpansive mappings, the space of all nonexpansive set-valued mappings) in the sense that its complement is σ-porous. All presented applications are known or are strengthenings of known results, however, they were proved separately by different mathematicians.

17. **Shakro Tetunashvili** (A. Razmadze Mathematical Institute, Tbilisi, Georgia), *On sums of convergent trigonometric series.* The aim of our talk is to present some properties of sums of convergent trigonometric series. One such property is: if a trigonometric series everywhere converges to a finite function $f(x)$, then for any point x_0 where exist one-sided limits $f(x_0+)$ and $f(x_0-)$, holds $2f(x_0) = f(x_0+) + f(x_0-)$. This statement is valid for both multi and one dimensional trigonometric series.
(This contribution was presented by V. Kokilashvili.)

18. **Jan Vybíral** (Johann Radon Institute for Computational and Applied Mathematics, Linz, Austria), *Average best m-term approximation.* The concept of best m-term approximation was introduced couple of decades ago and since then found many applications in approximation theory, image processing and other fields. We introduce and study the basic properties of the notion of average best m-term approximation.

19. **Ahmed I. Zayed** (DePaul University, Chicago, Illinois, USA), *Shift-invariant and sampling spaces in the fractional Fourier transform domain.* Shift-invariant spaces play an important role in the construction of multiresolution analyses. Sampling spaces are special cases of Shift-invariant spaces, in which functions are determined by their values on an infinite but discrete set of points. In this talk we first introduce the factional Fourier transform and some of its applications in optics, and then proceed to discuss properties of shift-invariant and sampling spaces in the fractional Fourier transform domain.

20. **Yuan Zhou** (University of Jyväskyla, Jyväskyla, Finland), *Pointwise characterizations of Besov and Triebel-Lizorkin spaces and quasiconformal mappings.* The classical Besov spaces $\dot{B}^s_{p,q}$ and Triebel-Lizorkin spaces $\dot{F}^s_{p,q}$ are characterized in terms of pointwise inequalities, for all $s \in (0,1)$ and $p, q \in (n/(n+s), \infty]$, both in $\mathbb{R}^n$ and in the metric measure spaces enjoying the doubling and reverse doubling properties. Applying this characterization, it is proved that quasiconformal mappings preserve $\dot{F}^s_{n/s,q}$ on $\mathbb{R}^n$ for all $s \in (0,1)$ and $q \in (n/(n+s), \infty]$. A metric measure space version of the above morphism property is also established.
(Joint work with P. Koskela and D. Yang.)