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A note on Buczolich's solution of the Weil gradient problem: a construction based on an infinite game

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1999

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2000

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2001

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[MATH-KMA-2001/61] Jiří Jelínek: *Equality of two diffeomorphism invariant Colombeau algebras.*

[MATH-KMA-2001/62] Ondřej Kalenda, Kenneth Kunen: *On subclasses of weak Asplund spaces.*

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2002

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2003

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[MATH-KMA-2003/101] Petr Holický, Tamás Keleti: *Borel classes of sets of extreme and exposed points in R^n.*

[MATH-KMA-2003/102] Robert Černý: *Local Monotonicity of Measures supported by Graphs of Convex Functions.*

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[MATH-KMA-2003/112] Eva Fašangová, Pedro J. Miana: *Spectral mapping inclusions for the Phillips functional calculus in Banach spaces and algebras.*

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[MATH-KMA-2003/115] Jiří Spurný: *The Dirichlet problem for Bair–one functions.*

[MATH-KMA-2003/116] Jan Čerych: *A characterization of essential sets of function algebras.*

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[MATH-KMA-2003/119] Miroslav Zelený, Luděk Zajíček: *Inscribing compact non-σ-porous sets into analytic non-σ-porous sets.*

[MATH-KMA-2003/120] David Pavlica: *On the points of non-differentiability of convex functions.*

[MATH-KMA-2003/121] Miroslav Zelený: *An absolutely continuous function with non-σ-porous graph.*

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2004

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[MATH-KMA-2004/134] Petr Holický, Luděk Zajíček, Miroslav Zelený: *A remark on a theorem of Solecki.*

[MATH-KMA-2004/135] Jakub Duda: *On a decomposition of superreflexive spaces.*

[MATH-KMA-2004/136] Jakub Duda: *Curves with finite turn.*

[MATH-KMA-2004/137] Jan Kališ: *On Whitney sets and their generalization.*

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[MATH-KMA-2004/141] Miroslav Zelený: *Descriptive properties of σ-porous sets.*

[MATH-KMA-2004/142] María J. Carro, Amiran Gogatishvili, Joaquim Martín, Luboš Pick: *Functional properties of rearrangement invariant spaces defined in terms of oscillations.*

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[MATH-KMA-2004/146] Petr Holický: *Borel sets with countable sections for nonseparable spaces.*

[MATH-KMA-2004/147] Jiří Spurný: *On lattice structure of the space of pointwise limits of harmonic functions.*

[MATH-KMA-2004/148] Jiří Spurný: *The weak Dirichlet problem for Baire functions.*

[MATH-KMA-2004/149] M. Bulíček, J. Málek, D. Pražák: *On the dimension of the attractor for a class of fluids with pressure dependent viscosities.*

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[MATH-KMA-2004/153] Libor Veselý, Luděk Zajíček: *On connection between d.c. mappings and convex operators.*

2005

[MATH-KMA-2005/154] Petr Holický a Václav Komínek: *Descriptive properties of mappings between nonseparable Luzin spaces.*

[MATH-KMA-2005/155] Jiří Spurný: *Affine Baire-one functions on compact convex sets.*

[MATH-KMA-2005/156] Jakub Duda: *On metric and w^*-differentiability of pointwise Lipschitz mappings.*

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[MATH-KMA-2005/158] David Pavlica, Luděk Zajíček: *Morse-Sard theorem for d.c. functions and mappings on R^2.*

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[MATH-KMA-2005/160] Jan Malý, Miroslav Zelený: *A note on Buczolich's solution of the Weil gradient problem: a construction based on an infinite game.*