David Pavlica

A d.c. C^1 function need not be the difference of convex C^1 functions

Preprint No. MATH-KMA-2004/138

June 2004

Submitted to *Comment. Math. Univ. Carolinae*

Department of Mathematical Analysis, Faculty of Mathematics and Physics,
Charles University, Sokolovská 83, 186 75 Praha 8, Czech Republic

http://www.karlin.mff.cuni.cz/kma-preprints/

Cover print by Reprostředisko, Malostranské nám. 25, Praha 1 - Malá Strana
Design by Mirko Rokyta, picture of snowflake by Ondřej Kalenda
Current series administrator: Mirko Rokyta

List of preprints released in this series

All preprints available on http://www.karlin.mff.cuni.cz/kma-preprints/
E-mail contact to series administrator: kma-prep@karlin.mff.cuni.cz

1999

[KMA-1999-01] Miroslav Zelený, Jan Pelant: *The structure of the σ-ideal of σ-porous sets.*
[KMA-1999-02] Ondřej Kalenda: *Valdivia compacta and equivalent norms.*
[KMA-1999-03] Ondřej Kalenda: *Embedding the ordinal segment $[0,\omega_1]$ into continuous images of Valdivia compacta.*
[KMA-1999-04] Jan Kolář, Jaroslav Lukeš: *Simultaneous solutions of the weak Dirichlet problem.*
[KMA-1999-05] Bohuslav Balcar, Miroslav Hušek: *Sequential continuity and submeasurable cardinals.*
[KMA-1999-06] Mirko Rokyta: *On the solvability of a nonlinear discrete problem corresponding to a higher-order finite volume approximation in 2D.*
[KMA-1999-07] Jan Čerych: *Pervasive algebras on planar compacts.*
[KMA-1999-08] Petr Holický, Miroslav Zelený: *Arsenin - Kunugui theorem and weak forms of Borel bimeasurability.*
[KMA-1999-09] Petr Holický: *Generalized analytic spaces, completeness and fragmentability.*
[KMA-1999-10] David E. Edmunds, Ron Kerman, Luboš Pick: *Optimal Sobolev imbeddings involving rearrangement-invariant quasinorms.*
[KMA-1999-11] A. Cianchi, R. Kerman, B. Opic, L. Pick: *A sharp rearrangement inequality for fractional maximal operator.*
[KMA-1999-12] Ondřej Kalenda: *Valdivia compacta and subspaces of $C(K)$ spaces.*
[KMA-1999-13] David Preiss, Luděk Zajíček: *On Dini and approximate Dini derivates of typical continuous functions.*
[KMA-1999-14] Ondřej Kalenda: *Valdivia compacta and biduals of Asplund spaces.*
[KMA-1999-15] Janusz J. Charatonik, Pavel Pyrih: *On piecewise confluent mappings.*
[KMA-1999-16] Janusz J. Charatonik, Pavel Pyrih: *Atomic mappings can spoil lightness of open mappings.*
[KMA-1999-17] Miroslav Zelený: *Remark on Debs – Saint-Raymond theorem.*

2000

[KMA-2000-18] Ralph Chill, Eva Fašangová: *Equality of two spectra arising in harmonic analysis and semigroup theory.*
[KMA-2000-19] Jan Malý: *Sufficient conditions for change of variables in integral.*
[KMA-2000-20] Pavel Pyrih: *An example for mappings related to semi-confluence.*
[KMA-2000-21] Pavel Pyrih, David Opěla, Robert Šámal: *An example of openly minimal dendrite.*
[KMA-2000-22] Jiří Spurný: *On the Dirichlet problem for functions of the first Baire class.*
[KMA-2000-23] Pavel Pyrih: *An example of self-homeomorphic dendrite not strongly self-homeomorphic.*
[KMA-2000-24] Pavel Pyrih, Tomáš Bárta, David Opěla, Pavel Růžička, Robert Šámal: *Irreducible mapping and the lightness of open mappings.*
[KMA-2000-25] Amiran Gogatishvili, Luboš Pick: *Duality principles and reduction theorems.*
[KMA-2000-26] David Opěla, Pavel Pyrih, Robert Šámal: *Chain of dendrites openly unbounded from bellow.*
[KMA-2000-27] Petr Holický, Václav Komínek: *Two remarks on the structure of the set of exposed and extreme points.*
[KMA-2000-28] David Preiss, Luděk Zajíček: *Directional derivatives of Lipschitz functions.*
[KMA-2000-29] Ondřej Kalenda: *A characterization of weakly Lindelöf determined Banach spaces.*
[KMA-2000-30] Jakub Duda: *On inverses of δ-convex mappings.*

In May 2000 this preprint series has been incorporated into a broader one of the entire School of Mathematics, Faculty of Mathematics and Physics, Charles University, and therefore the preprint numbering has been changed.

[MATH-KMA-2000/31] P. Gurka, F. J. Martín-Reyes, P. Ortega, L. Pick, M. D. Sarrión, A. de la Torre: *Good and bad measures.*
[MATH-KMA-2000/32] Robert Černý, Jan Malý: *Counterexample to lower semicontinuity in Calculus of Variations.*
[MATH-KMA-2000/33] Jan Malý, Luboš Pick: *An elementary proof of sharp Sobolev embeddings.*
[MATH-KMA-2000/34] Petr Holický, Luděk Zajíček: *Nondifferentiable functions, Haar null sets and Wiener measure.*

[MATH-KMA-2000/35] Jaroslav Milota: *Continuous semigroups and evolution equations.*

[MATH-KMA-2000/36] Jan Malý, Luboš Pick: *The sharp Riesz potential estimates in metric spaces.*

[MATH-KMA-2000/37] Mirko Rokyta: *A suitable replacement of the BV condition for finite volume schemes on unstructured grids.*

[MATH-KMA-2000/38] Jan Rychtář: *Uniformly Gâteaux differentiable norms in spaces with unconditional basis.*

[MATH-KMA-2000/39] Ondřej Kalenda: *Note on Markushevich bases in subspaces and quotients on Banach spaces.*

[MATH-KMA-2000/40] M. Carro, L. Pick, J. Soria, V.D. Stepanov: *On embeddings between classical Lorentz spaces.*

[MATH-KMA-2000/41] Eva Fašangová: *A Banach algebra approach to the weak spectral mapping theorem for $\mathcal{C}_0$-groups.*

2001

[MATH-KMA-2001/42] Jakub Duda, Libor Veselý, Luděk Zajíček: *On D.C. functions and mappings.*

[MATH-KMA-2001/43] W. D. Evans, Bohumír Opic, Luboš Pick: *Real interpolation with logarithmic functors.*

[MATH-KMA-2001/44] Robert Černý, Jan Malý: *Yet more counterexample to lower semicontinuity in Calculus of Variations.*

[MATH-KMA-2001/45] Wolfgang Desch, Eva Fašanga, Jaroslav Milota, Wilhelm Schappacher: *Stress boundary value problem in linear viscoleasticity.*

[MATH-KMA-2001/46] Ondřej Kalenda: *A new Banach space with Valdivia dual unit ball.*

[MATH-KMA-2001/47] Petr Holický, Roman Kotecký, Miloš Zahradník: *Phase diagram of horizontally invariant Gibbs states for lattice models.*

[MATH-KMA-2001/48] Stanislav Hencl: *On the notions of absolute continuity for functions of several variables.*

[MATH-KMA-2001/49] Jaroslav Lukeš, Jan Malý, Ivan Netuka, Michal Smrčka, Jiří Spurný: *On approximation of affine Baire-one functions.*

[MATH-KMA-2001/50] Wolfgang Desch, Eva Fašanga, Jaroslav Milota, Wilhelm Schappacher: *Riccati operators in non-reflexive spaces.*

[MATH-KMA-2001/51] Petr Holický, Václav Komínek: *On projections of nonseparable Souslin and Borel sets along separable spaces.*

[MATH-KMA-2001/52] Jan Lang: *Improved estimates for the approximation numbers of Hardy-type operators.*

[MATH-KMA-2001/53] Luboš Pick, Michael Růžička: *An example of a space $L^{p(x)}$ on which the Hardy–Littlewood maximal operator is not bounded.*

[MATH-KMA-2001/54] Amiran Gogatishvili, Bohumír Opic, Luboš Pick: *Weighted inequalities for Hardy-type operators involving suprema.*

[MATH-KMA-2001/55] Ondřej Kalenda: *M-bases in spaces of continuous functions on ordinals.*

[MATH-KMA-2001/56] Stanislav Hencl, Jan Malý: *Mappings of finite distortion: Hausdorff measure of zero sets.*

[MATH-KMA-2001/57] András Bátkai, Eva Fašanga: *The spectral mapping theorem for Davies' functional calculus.*

[MATH-KMA-2001/58] Jan Rychtář: *Renormings of $C(K)$ spaces.*

[MATH-KMA-2001/59] Ondřej Kalenda: *A weak Asplund space whose dual is not Stegall.*

[MATH-KMA-2001/60] Jiří Jelínek: *On introduction of two diffeomorphism invariant Colombeau algebras.*

[MATH-KMA-2001/61] Jiří Jelínek: *Equality of two diffeomorphism invariant Colombeau algebras.*

[MATH-KMA-2001/62] Ondřej Kalenda, Kenneth Kunen: *On subclasses of weak Asplund spaces.*

[MATH-KMA-2001/63] Andrea Cianchi, Luboš Pick: *Sobolev embeddings into spaces of Campanato, Morrey and Hölder type.*

[MATH-KMA-2001/64] Eva Fašanga: *Spectral mapping theorems and spectral space–independence.*

[MATH-KMA-2001/65] Miroslav Zelený: *An example of $\mathcal{C}^{1,1}$ function, which is not a d.c. function.*

[MATH-KMA-2001/66] Eva Matoušková, Miroslav Zelený: *A note on intersections of non-Haar null sets.*

[MATH-KMA-2001/67] Stanislav Hencl: *Measures of non-compactness of classical embeddings of Sobolev spaces.*

[MATH-KMA-2001/68] Jan Malý, David Swanson, William P. Ziemer: *The coarea formula for Sobolev mappings.*

[MATH-KMA-2001/69] W.Desch, E. Fašanga, J.Milota: *Unbounded observers and Riccati operators in nonreflexive spaces.*

[MATH-KMA-2001/70] Amiran Gogatishvili, Luboš Pick: *Discretization and anti-discretization of rearrangement-invariant norms.*

2002

[MATH-KMA-2002/71] Petr Holický, David Preiss: *Rearrangements of functions on totally disconnected groups.*

[MATH-KMA-2002/72] Petr Holický, Jiří Spurný: *Perfect images of absolute Souslin and absolute Borel Tychonoff spaces.*

[MATH-KMA-2002/73] Jan Malý: *Coarea properties of Sobolev functions.*

[MATH-KMA-2002/74] Jan Malý: *Wolf potential estimates of superminimizers of Orlicz type Dirichlet integrals.*

[MATH-KMA-2002/75] Ondřej Kalenda: *On the class of continuous images of Valdivia compacta.*

[MATH-KMA-2002/76] Marián Fabian, Ondřej Kalenda, Jan Kolář: *Filling analytic sets by the derivatives of C^1-smooth bumps.*

[MATH-KMA-2002/77] Aleš Nekvinda, Luděk Zajíček: *Gâteaux differentiability of Lipschitz functions via directional derivatives.*

[MATH-KMA-2002/78] Jaroslav Lukeš, Ivan Netuka, Jiří Veselý: *Choquet's theory and the Dirichlet problem.*

[MATH-KMA-2002/79] Stanislav Hencl: *A sharp form of an embedding into exponential and double exponential spaces.*

[MATH-KMA-2002/80] Jiří Spurný: *Representation of abstract affine functions.*

[MATH-KMA-2002/81] M. E. Mera, M. Morán, D. Preiss, L. Zajíček: *Porosity, σ-porosity and measures.*

[MATH-KMA-2002/82] Sophie Grivaux, Jan Rychtář: *Invariant subspaces of X^{**} under the action of biconjugates.*

[MATH-KMA-2002/83] Petr Hájek, Jan Rychtář: *On a problem of Borwein.*

[MATH-KMA-2002/84] Petr Holický, Jan Pelant: *Internal descriptions of absolute Borel classes.*

[MATH-KMA-2002/85] Stanislav Hencl: *Explicit upper bound for entropy numbers.*

[MATH-KMA-2002/86] Jan Kolář, Jan Kristensen: *Gradient ranges of bumps on the plane.*

[MATH-KMA-2002/87] Aleš Nekvinda, Luděk Zajíček: *Extensions of real and vector functions of one variable which preserve differentiability.*

[MATH-KMA-2002/88] Stanislav Hencl: *Notes on absolutely continuous functions of several variables.*

[MATH-KMA-2002/89] Stanislav Hencl: *Absolutely continuous functions of several variables and quasiconformal mappings.*

[MATH-KMA-2002/90] Michal Johanis: *Approximation of Lipschitz mappings.*

[MATH-KMA-2002/91] Jaroslav Lukeš, Ivan Netuka: *Extreme harmonic functions on a ball.*

[MATH-KMA-2002/92] M. Cwikel, A. Kamińska, L. Maligranda, L. Pick: *Are generalized Lorentz "spaces" really spaces?*

[MATH-KMA-2002/93] Ron Kerman, Luboš Pick: *Optimal Sobolev imbeddings.*

2003

[MATH-KMA-2003/94] Jan Lang, Aleš Nekvinda, Luboš Pick: *Boundedness and compactness of general kernel integral operators from a weighted Banach function space into L_∞.*

[MATH-KMA-2003/95] Jakub Duda: *On the size of the set of points where the metric projection exists.*

[MATH-KMA-2003/96] Luděk Zajíček, Miroslav Zelený: *On complexity of a σ-ideal of σ-$\mathbf{P}$-porous sets.*

[MATH-KMA-2003/97] Josef Daněček, Oldřich John, Jana Stará: *Interior $C^{1,\gamma}$-regularity for weak solutions of nonlinear second order elliptic systems.*

[MATH-KMA-2003/98] Ralph Chill, Eva Fašangová: *Convergence to steady states of solutions of semilinear evolutionary integral equations.*

[MATH-KMA-2003/99] Jan Rychtář: *Pointwise uniformly rotund norms.*

[MATH-KMA-2003/100] Petr Holický, Miklós Laczkovich: *Descriptive properties of the set of exposed points of compact convex sets in R^3.*

[MATH-KMA-2003/101] Petr Holický, Tamás Keleti: *Borel classes of sets of extreme and exposed points in R^n.*

[MATH-KMA-2003/102] Robert Černý: *Local Monotonicity of Measures supported by Graphs of Convex Functions.*

[MATH-KMA-2003/103] Jan Malý: *Coarea integration in metric spaces.*

[MATH-KMA-2003/104] J. Lukeš, T. Mocek, M. Smrčka, J. Spurný: *Choquet like sets in function spaces.*

[MATH-KMA-2003/105] Jiří Spurný: *F_σ–additive families and the invariance of Borel classes.*

[MATH-KMA-2003/106] Dalibor Pražák: *A necessary and sufficient condition for the existence of an exponential attractor.*

[MATH-KMA-2003/107] Petr Hájek, Michal Johanis: *Smooth approximations without critical points.*

[MATH-KMA-2003/108] Petr Hájek, Michal Johanis: *A characterization of reflexivity.*

[MATH-KMA-2003/109] Ondřej Kalenda, Matias Raja: *Descriptive properties of spaces of signed measures.*

[MATH-KMA-2003/110] Miroslav Hušek: *$C_p(X)$ in coreflective classes of locally convex spaces.*

[MATH-KMA-2003/111] Petr Holický, Jiří Spurný: *F_σ-mappings and the invariance of absolute Borel classes.*

[MATH-KMA-2003/112] Eva Fašangová, Pedro J. Miana: *Spectral mapping inclusions for the Phillips functional calculus in Banach spaces and algebras.*

[MATH-KMA-2003/113] Ondřej Kalenda: *Note on countable unions of Corson countably compact spaces.*

[MATH-KMA-2003/114] Stanislav Hencl, Jan Malý: *Absolutely continuous functions of several variables and diffeomorphisms.*

[MATH-KMA-2003/115] Jiří Spurný: *The Dirichlet problem for Bair–one functions.*

[MATH-KMA-2003/116] Jan Čerych: *A characterization of essential sets of function algebras.*

[MATH-KMA-2003/117] Jiří Jelínek: *A discontinuous function with a connected closed graph.*

[MATH-KMA-2003/118] Dalibor Pražák: *On the dimension of the attractor to the wave equation with nonlinear damping.*

[MATH-KMA-2003/119] Miroslav Zelený, Luděk Zajíček: *Inscribing compact non-σ-porous sets into analytic non-σ-porous sets.*

[MATH-KMA-2003/120] David Pavlica: *On the points of non-differentiability of convex functions.*

[MATH-KMA-2003/121] Miroslav Zelený: *An absolutely continuous function with non-σ-porous graph.*

[MATH-KMA-2003/122] Amiran Gogatishvili, Luboš Pick: *Embeddings and duality theorems for weak classical Lorentz spaces.*

2004

[MATH-KMA-2004/123] Petr Holický: *Extension of Borel measurable maps and ranges of Borel bimesurable maps.*

[MATH-KMA-2004/124] Jiří Spurný: *Affine Baire–one functions on Choquet simplexes.*

[MATH-KMA-2004/125] Luděk Zajíček: *On $\sigma-$porous sets in abstract spaces (a partial survey).*

[MATH-KMA-2004/126] R. Chill, E. Fašangová, G. Metafune, D. Pallara: *The sector of analycity of the Ornstein-Uhlenbeck semigroup on L^p spaces with respect to invariant measure.*

[MATH-KMA-2004/127] Petr Hájek, Michal Johanis: *Isomorphic Embeddings and Harmonic Behaviour of Smooth Operators.*

[MATH-KMA-2004/128] Tomáš Bárta: *On a Viscoelastic Problem.*

[MATH-KMA-2004/129] Étienne Matheron, Miroslav Zelený: *Rudin-like sets and hereditary families of compact sets.*

[MATH-KMA-2004/130] Dalibor Pražák: *Reducing the 2d Navier-Stokes equations to a system of delayed ODEs.*

[MATH-KMA-2004/131] Luděk Zajíček, Miroslav Zelený: *Inscribing closed non-σ-lower porous sets into Suslin non-σ-lower porous sets.*

[MATH-KMA-2004/132] Petr Dostál, Jaroslav Lukeš, Jiří Spurný: *Measure convex and measure extremal sets.*

[MATH-KMA-2004/133] Ondřej Kalenda, Jiří Spurný: *Extending Baire-one functions on topological spaces.*

[MATH-KMA-2004/134] Petr Holický, Luděk Zajíček, Miroslav Zelený: *A remark on a theorem of Solecki.*

[MATH-KMA-2004/135] Jakub Duda: *On a decomposition of superreflexive spaces.*

[MATH-KMA-2004/136] Jakub Duda: *Curves with finite turn.*

[MATH-KMA-2004/137] Jan Kališ: *On Whitney sets and their generalization.*

[MATH-KMA-2004/138] David Pavlica: *A d.c. C^1 function need not be the difference of convex C^1 functions.*