Some Trends in Algebra '07

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A B S T R A C T S O F T A L K S

Semigroup Crossed products and Toeplitz algebras

Mamoon Ahmed

The theory of crossed products of C^*-algebras by semigroups of endomorphisms has been developing rapidly. This theory is a generalization of the theory of crossed products of C^*-algebras by semigroups of automorphisms, which is an interesting area of the modern theory of operator algebras. The significance of that theory have led many authors to consider more general cases of crossed products of C^*-algebras by semigroups of automorphisms such as Murphy and the crossed products by semigroups of endomorphisms suchas Raeburn. I will talk here about crossed products by semigroups of endomorphisms, actions of general semigroups, namely the positive cone G_+ of a partially ordered discrete abelian group G and with C^*-algebras A which might not have an identity. I have shown that we can realize the C^*-algebra $B_{(G/H)_+} \times_\beta G_+$ as the induced C^*-algebra $Ind_{H^\perp}^{\widehat{G}} \left(B_{(G/H)_+} \times_\tau (G/H)_+ \right)$.

On the normality of t-Cayley hypergraphs on abelian groups

Mehdi Alaeiyan

A t-Cayley hypergraph $X = t\text{-}Cay(G, S)$ is normal for G if the right regular representation $R(G)$ of G is normal in the full automorphism group $Aut(X)$. In this paper, we give a sufficient condition for the normality of t-Cayley hypergraphs of abelian groups, where $|S| \leq 4$.

MSC: 05C25, 20B25
Keywords: hypergraphs, t-Cayley hypergraph, normal t-Cayley hypergraph.

Warfield duality for self-small mixed groups

Simion Breaz

We consider a self-small group A of torsion free rank 1 and the pair of adjoint contravariant functors $W(-) = \mathrm{Hom}(-, A) : Ab \to Ab : \mathrm{Hom}(-, A) = W(-)$ (Ab denotes the category of all abelian groups), together with the induced arrows $\delta_C : C \to W^2$, $\delta(c) = [f \mapsto f(c)]$. We study A-W-reflexive abelian groups, i.e. those groups C such that δ_C is an isomorphism.

Rings with finite Gorenstein global dimension

Sergio Estrada

We find new classes of non noetherian rings which have the same homological behavior that Gorenstein rings.

Differential polynomial rings of triangular matrix rings

Hoger Ghahramani

Let R, S be rings and M be an $(R, S)-$ bimodule. In this paper we characterize the derivations of the Generalized matrix rings $T = \begin{pmatrix} R & M \\ 0 & S \end{pmatrix}$ in terms of derivations of the rings R, S and the generalized derivations of M and inner derivations. Using the result we provide a triangular representation of differential polynomial ring $T[\theta, d]$ in terms of the triangular matrix ring:
$\begin{pmatrix} R & M \\ 0 & S \end{pmatrix}[\theta : d] \cong \begin{pmatrix} R[x; \delta_R] & M[x, y; \tau] \\ 0 & S[y; \delta_S] \end{pmatrix}$.

Homological characterizations of Prüfer-like rings

Sarah Glaz

This talk will survey the results obtained by the speaker and others in the ongoing investigation into the nature of various Prüfer-like properties of rings. Throughout the investigation two features stood out. First: the relations among the various Prüfer-like properties of a ring depend heavily on the Prüfer-like properties of the total rings of quotients involved. Second: these properties can best be investigated through the clarification of the weak global dimensions, and the finitistic projective dimensions of the rings involved. We will discuss the relations among the various Prüfer-like properties, characterizations of these properties, examples, counterexamples, and conjectures, highlighting the two features mentioned above.

Finitistically quasi-tilted rings

Enrico Gregorio

A right noetherian ring R is said to be *finitistically quasi-tilted* if there exists a split torsion pair $(\mathcal{X}_0, \mathcal{Y}_0)$ in mod- such that

(1) every module in $\mathcal{Y}_0$ has projective dimension ≤ 1;
(2) $R \in \mathcal{Y}_0$.

Many results about quasi-tilted artin algebras can be extended to finitistically quasi-tilted rings. An example will be given of a finitistically quasi-tilted ring which is not tilted (i.e., it is not the endomorphism ring of any tilting module over any ring). A right noetherian ring R of global dimension ≤ 2 is *finitistically almost hereditary* if every finitely generated indecomposable right R-module either has projective dimension ≤ 1 or injective dimension ≤ 1. Every finitistically quasi-tilted ring is finitistically almost hereditary and the converse implication holds when the ring is an artin algebra, while it is not known in the noetherian case.

Decompositions of modules and generalized Mittag-Leffler modules

Pedro Antonio Guil Asensio

Let κ be an infinite regular cardinal and M, a κ-presentable module. The category $\kappa - lim\, add[M]$ consisting on the κ-directed limits of direct summands of finite direct sums of copies of M is a κ-accessible category in the sense of Adamek and Rosicky. We give a notion of pure-semisimplicity for this category and show that, if this category is not pure-semisimple, then it is possible to construct a non-trivial κ-separable module associated to it. This construction suggest that these modules control the pure-semisimplicity of these categories. Moreover, these modules can be seen as transfinite generalizations of Strict Mittag-Leffler modules, in the sense of Grusson and Reynaud. Our results require the assumption of the Generalized Continuum Hypothesis and they extend a construction of separable modules given by Eklof.

Joint work with M.C. Izurdiaga and B. Torrecillas.

On prime and coprime modules and preradicals

Josef Jirásko

We investigate various types of generalized prime and coprime modules and preradicals corresponding to them.

Model Theory of modules and Diophantine Geometry: possible interactions on Lie algebras

Sonia L'Innocente

This report aims at illustrating the main results of two joint works, the former with Angus Macintyre [4] and the latter with Ivo Herzog [2], both related to a model theoretic investigation of certain representations over sl_2k –the Lie algebra of trace zero 2×2 matrices over an algebraically closed field k of characteristic zero– and inspired by Herzog's paper [1] showing several remarkable results about the ring of definable scalars U_k' of finite-dimensional representations of the enveloping algebra U_k of sl_2k.

In [4], we investigate the theory of all pseudo-finite dimensional representations of U_k, (meaning infinite dimensional representations satisfying the same first order sentences as finite dimensional representations of U_k). Our principal contribution is to bring out a connection to fundamental problems in the diophantine geometry of curves. We show that the problem of decidability is related to an construction of the ring U_k'. We are able to give a recursive presentation of U_k', building it from U_k in stages and assuming some plausible conjectures about the decision problem for integer points on curves. We are obliged to describe the structure of the sets $\{\lambda : V_\lambda \models \Phi\}$, where V_λ is a U_k-module of dimension (over k) $\lambda + 1$ and where Φ a sentence of the language of U_k-modules. In this paper we bring out some basic new information about the case when Φ concerns the nontriviality of certain kernels. This is where diophantine geometry is relevant.

In [2], we prove the analogous results of [1] by enlarging the classical case of U_k to the quantum case of the quantized enveloping algebra U_q where the parameter q is not a root of unity. Let $k_q[x, y]$ be the *quantum plane* [3], defined to be the free k-algebra $k\{x, y\}$ generated by x and y, modulo the relation $yx = qxy$, and regarded as a U_q-module. We show that in the language of U_q-modules, the ring of definable scalars of the quantum plane is a von Neumann regular epimorphic ring extension of the quantum group U_q.

We also discuss some open problems, involved by these works.

References

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[3] C. Kassel, *Quantum Groups,* Grad. Texts in Math. 155, Springer-Verlag, 1995.

[4] S. L'Innocente, A. Macintyre, Towards decidability of the theory of pseudo-finite dimensional representations of sl_2k, Part I, *Accepted by Fundamenta Mathematicae, a special volume dedicated to the memory of Andrzej Mostowski.*

Tilting and cotilting modules over 1-Gorenstein rings

David Pospíšil

Let R be a commutative 1-Gorenstein ring. Our main result characterizes all tilting and cotilting R-modules: up to equivalence: they are parametrized by subsets of the set of all prime ideals of height one. More precisely, every tilting (cotilting) R-module is equivalent to some Bass tilting (cotilting) module.

How could we classify big projectives over $Z[A_5]$

Pavel Příhoda

I will discuss one possible method for classification of nonfinitely generated projective modules over noetherian rings, co called fair-sized modules. This elementary technique is based on old results of Bass and Whitehead, and its says that over noetherian rings that satisfy certain condition on two-sided ideals, countably generated projective modules are determined by a pair (I, P), where I is an idempotent ideal and P is a finitely generated projective R/I-module. The particular cases of generalized Weyl algebras, $Z[A_5]$ and $U(sl_2(C))$ will be considered in order to explain the limits of this method.

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Generalized lattices from point of view of projective modules

Gennadi Puninski

Suppose that D is a Dedikind domain with the quotient field K, and let Λ be a D-order in a semisimple finite dimensional K-algebra. A Λ-module M is said to be a *generalized lattice* if M is projective as a D-module.

Very recently Rump found a criterion when every generalized lattice over an order of finite lattice type is a direct sum of lattices. For instance this is the case for integral group ring of a finite group of order p or p^2.

In this talk we will explain how to apply Prihoda's classification of projective modules over noetherian rings to classify generalized lattices over some orders of a finite lattice type.

CLP and related problems

Pavel Růžička

We will present the main ideas of the recent solution of the congruence lattice problem and some related results and questions.

Deconstruction of cotorsion pairs and telescope conjecture

Jan Šaroch

After short introduction to the topic, a useful method for analyzing cotorsion pairs of modules will be presented: so-called "deconstruction" of cotorsion pairs. Several sufficient conditions for deconstruction and related techniques will be discussed; together with some applications. The most recent one concerns an interesting open problem coming originally from algebraic topology, the Telescope conjecture (for module categories).

Structure problems for Ext and Bext

Lutz Strüngmann

In 1977 Saharon Shelah solved the well-known Whitehead problem by showing that it is undecidable in ordinary set-theory ZFC whether or not every abelian group G satisfying $\mathrm{Ext}_{\mathbf{Z}}(G,\mathbf{Z}) = \{0\}$ has to be free. However, this did not clarify the structure of $\mathrm{Ext}_{\mathbf{Z}}(G,\mathbf{Z})$ for torsion-free abelian groups - a problem which has received much attention since then. Easy arguments show that $\mathrm{Ext}_{\mathbf{Z}}(G,\mathbf{Z})$ is always a divisible group for every torsion-free group G. Hence it is of the form

$$\mathrm{Ext}_{\mathbf{Z}}(G,\mathbf{Z}) = \bigoplus_{p\in\Pi} \mathbf{Z}(p^\infty)^{(\nu_p)} \oplus \mathbf{Q}^{(\nu_0)}$$

for some cardinals ν_p, ν_0 $(p \in \Pi)$ which are uniquely determined. The obvious question that arises is which sequences $(\nu_0, \nu_p : p \in \Pi)$ of cardinals can appear as the cardinal invariants of $\mathrm{Ext}_{\mathbf{Z}}(G,\mathbf{Z})$ for some (which) torsion-free abelian group? We will give a complete characterization assuming Goedel's constructible universe L plus there is no weakly compact cardinal. Moreover, we shall consider different models of ZFC in which we can reach the borderline. For instance, we use the existence of a super compact cardinal to show that there is a model of set theory in which the cardinal invariants of any torsion-free group are as maximal as possible. Other models are *close* to ZFC but still we have a strange behavior of those invariants. All of this is joint work with Saharon Shelah. Last but not least we will consider related structure problems for the functor $Bext$ which plays an important role in the theory of infinite rank Butler groups. This is work jointly done with Rüdiger Göbel and Nicole Hülsmann.

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The Telescope Conjecture and vanishing of the transfinite radical

Jan Šťovíček

This talk will build on results presented by Jan Šaroch. I will explain a solution to the Telescope Conjecture for a certain class of artin algebras, namely those which have no non-trivial idempotent ideals inside the radical of the category of finitely generated modules. This includes for example domestic standard selfinjective algebras and domestic special biserial algebras.

I will discuss the concept of an ideal of an additive category, the structure of idempotent ideals of categories of finitely generated modules, and finally the connection between the idempotent ideals and the Telescope Conjecture.

The model theoretic complexity of modules over commutative noetherian rings

Carlo Toffalori

I consider modules over commutative noetherian rings R and I discuss in this framework several questions linking model theory and algebra (such as decidability, existence of superdecomposable pure injective modules and so on).

Some tilting modules for 2-Gorenstein rings

Jan Trlifaj

In [1] a construction of tilting modules over (commutative) 1–Gorenstein rings R was presented using pullbacks of the minimal injective coresolution of the regular module. These pullbacks have recently been shown to yield all tilting R–modules up to equivalence, [2]. The situation for 2–Gorenstein rings is much more complex: In my talk I will present several 2–dimensional variants of the pullback construction, and prove that they yield non–equivalent 2–tilting modules.

[1] L.Angeleri Hügel, D.Herbera, and J.Trlifaj: "Tilting modules and Gorenstein rings", Forum Math. 18(2006), 211-229.

[2] J.Trlifaj and D.Pospíšil: "Tilting and cotilting classes over Gorenstein rings", preprint (2007).

Self-small modules over perfect rings

Jan Žemlička

A right R-module M is said to be self-small if the covariant functor $\mathrm{Hom}(M, -)$ commutes with direct sums of copies of M. Clearly, every finitely generated module is self-small, nevertheless, examples of infinitely generated self-small modules occur over many various rings, including Artin algebras. We will discuss the existence of such examples of modules over perfect rings and over rings close to perfect.