Some Trends in Algebra '03
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ABSTRACTS OF TALKS

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CONSTRUCTING TILTING MODULES OF PROJECTIVE DIMENSION ONE OVER COMMUTATIVE RINGS

Lidia Angeleri Hügel

I will report on ongoing work with Dolors Herbera and Jan Trlifaj. We consider the localization R_S of a commutative ring R with respect to a submonoid S of the set of regular elements. We focus on the case that R_S has projective dimension at most one, using ideas developed by Hamsher and Matlis for domains. We then construct a tilting module whose tilting class is the class of S-divisible modules. Hereby we generalize known constructions of tilting modules over domains and over Gorenstein rings.

1-TILTING MODULES

Silvana Bazzoni

We consider a right 1-tilting module T over an arbitrary ring R. We will show that T is of countable type. Moreover, we will show that T is of finite type in case R is a Prüfer domain.

ON PRECOVER CLASSES

Ladislav Bican

Let $\mathcal{G}$ and $\mathcal{H}$ be abstract classes of modules. The class $\mathcal{H}$ is said to have the $\mathcal{G}$-property, if to each infinite cardinal λ there exists a cardinal $\kappa > \lambda$ such that for every $F \in \mathcal{H}$ with $|F| \geq \kappa$ and every its submodule K with $|F/K| \leq \lambda$ there exists a submodule L of K such that $F/L \in \mathcal{G}$ and $|F/L| < \kappa$. This condition is stronger than the condition (P) requiring $L \neq 0$ instead of $|F/L| < \kappa$, which was introduced and investigated in my previous paper with B. Torrecillas. In this work we are going to study the relations of this more general condition to the existence of precovers with respect to some classes of modules. As an application we obtain some sufficient conditions for the existence of σ-torsionfree precovers related to a given hereditary torsion theory σ for the category R-mod. This result is closely related to and in some sense extends my previous results of this kind.

SERIAL COALGEBRAS

Juan Cuadra Díaz

This is a joint work with Prof. Gómez-Torrecillas from the University of Granada (Spain).

In the last years different kind of coalgebras has been studied attending to some properties of its categories of comodules. For example, a coalgebra C is:

- *right semiperfect* if its category of right comodules has enough projective objects;

- *left quasi-coFrobenius* if every injective right comodule is projective;

- *hereditary* if quotients of injective comodules are injective;

- *left pure semisimple* if every left C-comodule is a direct sum of finite dimensional comodules.

In this talk we will continue this line of research by introducing serial coalgebras. A coalgebra C is called *right serial* if its right injective indecomposable comodules are uniserial (i.e. the lattice of subcomodules is a chain). A *serial* coalgebra is a right and left serial coalgebra. Examples of serial coalgebras will be provided as finite dual coalgebras of hereditary prime noetherian algebras. Several structure theorems and characterizations of serial coalgebras will be given.

ON LARGE SELF-ORTHOGONAL MODULES

Gabriella d'Este

We use finite dimensional algebras to construct quite different selforthogonal modules, of finite projective (resp. injective) dimension, large enough to satisfy a vanishing Hom and Ext condition without being tilting (resp. cotilting) modules.

COTORSION THEORIES AND COGALOIS GROUPS

Sergio Estrada

Suppose $\mathcal{F}$ is a class of objects which is closed under isomorphisms in an abelian category. If an object M admits an $\mathcal{F}$-cover, namely $\varphi : F \longrightarrow M$, we have the subgroup of the group of automorphisms of F consisting of all endomorphisms $f : F \longrightarrow F$ such that $\varphi \circ f = \varphi$. This is the coGalois group of the cover. For the case of torsion free covers over in [1] a topology for these groups is defined. With this topology we shall describe the compact coGalois groups by reducing the problem to consider torsion free covers of cotorsion modules having no nonzero divisible submodules.

[1] E. Enochs, J. R. García-Rozas, O. M. G. Jenda and L. Oyonarte, *Compact coGalois groups*, Math. Proc. Cambridge Phil. Soc. (2) **128** (2000), 233-244.

EQUIVALENCE AND DUALITY FOR MODULE CATEGORIES

Kent Fuller

I will discuss some highlights of our forthcoming book with R. R. Colby.

UNIQUELY TRANSITIVE ABELIAN GROUPS

Rüdiger Göbel

E. Farjoun raised the question about the existence of UT-groups. An abelian group A is UT if $\operatorname{Aut} A$ acts sharply transitive on its pure elements. In order to avoid trivial cases we assume that A is $\aleph_1$-free (all countable subgroups are free, so there are enough pure elements in A): $A =$ is a well-known example, the only one for some time. In a joint paper with S. Shelah we construct a proper class of UT-groups which are $\aleph_1$-free; to appear in the Venice proceedings.

GENERALIZED ARTIN ALGEBRAS

Enrico Gregorio

Many results holding for Artin algebras can be extended to algebras which are not subject to strong finiteness conditions, by using topological methods. We shall present some extensions for quasi-Frobenius algebras, Colby-Fuller duality and so on.

Σ-COTORSION RINGS AND MODULES

Pedro A. Guil Asensio

This is a joint work with I. Herzog. It is proved that a ring R is right perfect iff it is Σ-cotorsion as a module over itself. Several other conditions are shown to be equivalent. For example, that every pure submodule of a free right R-module is strongly pure-essential in a direct summand, or that the countable direct sum $\mathrm{C}(R_R)^{(\aleph_0)}$ of the cotorsion envelope of R_R is cotorsion.

If C_R is a flat Σ-cotorsion module, then C_R admits a decomposition into indecomposable modules with local endomorphism ring. The Jacobson radical $\mathcal{J}(S)$ of the endomorphism ring $S = \mathrm{End}(C)$ is characterized as the maximum ideal that acts locally T-nilpotently on C_R. If R is semilocal and $C = \mathrm{C}(R_R)$, then its radical consists of those endomorphisms $f : C \longrightarrow C$ whose image is contained in $C\mathcal{J}$.

FLAT AND PROJECTIVE MODULES

Dolors Herbera

Although there is a close relation between finitely generated projective right R-modules and finitely generated projective left R-modules given by the duality $\mathrm{Hom}_R(-, R)$, there does not seem to be such an evident relation between finitely generated flat right R-modules and finitely generated flat left R-modules. In recent work with A. Facchini and I. Sakhajev, we define an algebraic object that allows us to describe right flat modules and left flat modules at the same time. We call this object a net.

As an application of our theory, we study how projective modules over the ring $R/J(R)$ lift to projective or flat modules over R. For instance, we find that for a finitely generated projective right $R/J(R)$-module P, there exists a finitely generated flat *right* R-module M with $M/MJ(R)$ isomorphic to P if and only if there exists a projective *left* R-module P' with $P'/J(R)P'$ isomorphic to the dual $\mathrm{Hom}_{R/J(R)}(P, R/J(R))$ of P. This result extends and complements previous results obtained by I. Sakhajev and by H. Zoschinger.

The problem of lifting projective modules modulo the Jacobson radical arises naturally in case $R/J(R)$ is semisimple artinian, that is, when R is a semilocal ring. We will explain some examples and constructions for this case, the most interesting one being an example of a projective module with a semilocal endomorphism ring, so having a strong finiteness condition, that is not finitely generated.

GROUP ALGEBRAS IN WHICH EACH COMPLEMENT RIGHT IDEAL IS A DIRECT SUMMAND

Surender K. Jain

We will give introduction to rings with the property that each complement (= closed) right ideal is a direct summand, and then talk of recent work for prime, semiprime, semilocal group algebras of polycyclic-by-finite groups with this property.

ON n–FTF–RINGS

Josef Jirásko

The rings for which there is a hereditary torsion theory s_n for R-Mod such that a left R-module is s_n-torsionfree if and only if it embeds into a left n-flat R-module are characterized.

WAKAMATSU TILTING MODULES

Francesca Mantese

We study a generalization of tilting modules to modules of possibly infinite projective dimension, introduced by Wakamatsu. In particular, we characterize these modules in terms of suitable subcategories of finitely generated modules and in terms of cotorsion theories.

ADD(M) OF SERIAL MODULE OF FINITE GOLDIE DIMENSION OVER SERIAL RINGS

Pavel Příhoda

In this talk we prove that any direct summand of a serial module of finite Goldie dimension is serial. Using this "result" we give a description of direct sum decompositions in add(M) for a serial module M of finite Goldie dimension.

GALOIS THEORY OF TORSION-FREE ENDOMORPHISM RINGS

Phillip Schultz

Given a torsion–free abelian group G, I define Galois connections among the three lattices:

(1) Pure fully invariant subgroups of G;
(2) Torsion–free factor groups of G;
(3) Pure ideals of End(G);

These connections are used to describe the lattices of pure ideals and left and right annihilator ideals of the endomorphism ring of a completely decomposable group.

BRAUER-THRALL CONJECTURES, LARGE INDECOMPOSABLE MODULES, ENDO-WILDNESS AND THE PURE SEMISIMPLE PROBLEM

Daniel Simson

The existence of large not finitely generated modules over right artinian ring R is discussed in connection with the pure semisimple problem and the endo-wildness (resp. Endo-Wildness) of the category $\mathrm{Mod}(R)$ of right R-modules. In particular, the following two conjectures, related with the famous Brauer-Thrall Conjectures are discussed.

Conjecture 1_∞. *A right artinian ring R is either right pure semisimple, or else, for each infinite cardinal number λ, there exists an indecomposable right R-module of cardinality $\geq \lambda$.*

Conjecture 2_∞. *A right artinian (or right noetherian) ring R is either right pure semisimple or else, there exists a sequence $\lambda_1 > \cdots > \lambda_\eta > \cdots$ of infinite cardinals such that, for each η, the set of the isomorphism classes of the indecomposable right R-modules of cardinality λ_η has the cardinality $> \lambda_\eta$.*

The validity of Conjecture 1_∞ is proved for finite dimensional K-algebras R, over an algebraically closed field K, that are strongly simply connected, or loop finite.

Relations with endo-wildness, Corner-type endo-wildness and Kaplansky's Test Problems are discussed.

For details we refer to the paper:

D. Simson, *On large indecomposable modules and right pure semisimple rings*, Algebra and Discrete Math. **2** (2003), 93–117.

TORSION THEORY AND COMODULES

Robert Wisbauer

Let C be any coalgebra over a commutative ring R. Then the dual module $C^* = \mathrm{Hom}_R(C, R)$ allows for a ring structure. Under mild conditions on the R-module structure of C, the category $\mathbf{M}^C$ of right comodules over C can be considered as a (coreflexive) subcategory of the category of left C^*-modules C^*-Mod. In fact it can be identified with $\sigma[_{C^*}C]$, the full subcategory whose objects are submodules of C-generated modules.

In the talk it will be explained how torsion theory of modules over a ring can be applied to understand the structure of comodules of a coalgebra.

Reference. T. Breziński and R. Wisbauer, *Corings and Comodules*, London Math. Soc. LN 309, Cambridge University Press, September 2003

COMMUTATIVE SEMIARTINIAN RINGS

Jan Žemlička

The structure of a von Neumann regular semiartinian ring with primitive factors artinian (in particular of a commutative regular semiartinian ring) can be described by the invariant of dimension sequences. By similar means we study the structure of an arbitrary general commutative semiartinian ring, which is embeddable into a suitable product of local perfect rings.

ON THE NILPOTENCY OF 2-GENERATED SUPERALGEBRAS IN THE VARIETY OF NIL-ALGEBRAS OF DEGREE 5

Natalia Zhukavets

This is a joint work with I. Shestakov. Let $\mathcal{N}il_n$ be the variety of associative algebras over a field of characteristic zero defined by the identity $x^n = 0$. The classical Dubnov-Ivanov-Nagata-Higman theorem states that every algebra from $\mathcal{N}il_n$ is nilpotent of degree $2^n - 1$. Razmyslov (1974) improved the nilpotency index in this theorem to n^2. Kuzmin (1975) showed that the index cannot be less than $f(n) = \frac{n(n+1)}{2}$ and conjectured that the last number gives the exact estimate of nilpotency for the variety $\mathcal{N}il_n$.

It is easy to see that Kuzmin's conjecture is true for $n = 2, 3$. Vaughan-Lee (1993) applied to the problem the representation theory of symmetric groups and the superalgebra techniques, and proved by computer that the conjecture is also true for $n = 4$, that is, every algebra form $\mathcal{N}il_4$ is nilpotent of degree $f(4) = 10$.

We consider the conjecture for the next value $n = 5$. Let $\widetilde{\mathcal{N}il}_5$ be the variety of $\mathcal{N}il_5$-superalgebras. Following Vaughan-Lee, it is easy to see that in order to confirm Kuzmin's conjecture for $n = 5$, it suffices to prove that any superalgebra from $\widetilde{\mathcal{N}il}_5$ on $k \leq 5$ homogeneous generators is nilpotent of degree $f(5) = 15$.

Using the GAP computer package, we prove that this result is true for 2-generated superalgebras. Observe that the two computer programs that gave an implementation of the same algorithm as that of Vaughan-Lee ('Albert' (Iowa University), 'Malcev' (Sobolev Institute of Math.)) were not able to solve the case of 2-generator superalgebras in the variety $\widetilde{\mathcal{N}il}_5$ even in the easiest case of two even variables.