1. Test

Answers should be brief, clear and precise. Every correct answer is for 3 points.

1. Give two non-isomorphic examples of an algebraic function field over $\mathbb{F}_5$.

2.
What is a Weierstrass equation polynomial?

3. Find prime ideals $0 \neq P \subsetneq Q$ of the ring $\mathbb{F}_3[x, y]$.

4. What is m-weighted multiplicity μ of a polynomial $a(x, y)$?

5. Formulate The Weak Approximation Theorem.

6. Compute a negative part of a principal divisor $(x)_-$ of the AFF $K(x)$ over K.

7. What is Weil differntial?

8. Determine $l(W)$, $\deg(W)$ a $i(W)$ for a canonical divisor W.

9. Write an example of an AFF of genus 0.

10. Define the Picard group $P^0(L/K)$ of an AFF L over K.

2. Computations

2.1. For the Weierstrass polynomial $w = y^2 - y - (x^3 + x^2 + 1) \in \mathbb{F}_5[x, y]$ find a $\mathbb{F}_5$-equivalent short WEP.

6 points

2.2. Let w be a smooth WEP and L be an AFF over $\mathbb{F}_{32}$ given by $w(\alpha, \beta) = 0$. Compute $\deg(\alpha\beta)$, $\deg(\alpha\beta)_+$, and $\deg(\alpha^{-1})_-$.

7 points

2.3. Decide whether $\mathbb{F}_2(V_w)$ is an EFF, if for $w = y^2 + y + x^3 + 1 \in \mathbb{F}_2[x, y]$.

7 points

3. Proofs

3.1. If L is an AFF over K, $P \in \mathbb{P}_{L/K}$, prove that $\mathcal{O}_P$ is a uniquely defined discrete valuation ring and that $\deg P$ is finite.

20 points

3.2. Describe the structure of vector spaces of Weil differentials $\Omega_{L/K}$ and $\Omega_{L/K}(A)$ as subspaces of the space L.

20 points