

When is the heart of a t-structure a Grothendieck category?

Jan Šťovíček

Contact: `stovicek@karlin.mff.cuni.cz`, MFF UK

Abstract: I will discuss the problem of what conditions a t-structure on a triangulated category must satisfy for its heart to be a Grothendieck category. Of course, the trickiest part is to show that the heart is an (AB5) abelian category, i.e. that direct limits are exact.

I will discuss two approaches. One of them uses the abstraction of pure injectivity to abelian or triangulated categories. The idea is originally due to Ivo Herzog and was used in [5, 1] and more recently in [2]. The new phenomenon appearing in [2] is that pure-injective objects are discussed in a general Grothendieck category, without any assumption on local finite presentability.

The second approach, pursued in [4], is to study direct limits in the heart via homotopy colimits in the ambient triangulated category. For this to work, we need an enhancement of the triangulated category. We choose a minimalistic one where homotopy colimits exist and are well behaved—stable derivators [3].

References

- [1] Angeleri Hügel, L., Marks, F., Vitória, J., Torsion pairs in silting theory. *preprint*, arXiv:1611.08139.
- [2] Čoupek, P., Šťovíček, J., Cotilting sheaves on Noetherian schemes. *preprint*, arXiv:1707.01677.
- [3] Groth, M., Derivators, pointed derivators and stable derivators. *Algebr. Geom. Topol.* **13** (2013), 313–374.
- [4] Saorín, M., Virili, S., Šťovíček, J., When is the heart of a t-structure a Grothendieck category?, *preprint*.
- [5] Šťovíček, J., Derived equivalences induced by big cotilting modules. *Adv. Math.* **263** (2014), 45–87.