On a nice behaviour of Gorenstein injective and Gorenstein flat modules over a general ring

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Abstract: Let R be an arbitrary ring and let $\mathcal{GI}, \mathcal{GF}$ denote the classes of all Gorenstein injective, Gorenstein flat (right R-)modules, respectively. We show that the pairs $\mathfrak{GI} = ({}^{\perp}\mathcal{GI}, \mathcal{GI})$ and $\mathfrak{GF} = (\mathcal{GF}, \mathcal{GF}^{\perp})$ are complete cotorsion pairs, with the left-hand classes closed under $\varinjlim$. In particular, $\mathcal{GI}$ is an enveloping class and $\mathcal{GF}$ is a covering class of modules, by the classic result of Enochs.

The cotorsion pair $\mathfrak{GI}$ naturally induces the Gorenstein injective abelian model structure where all modules are cofibrant, modules from $\mathcal{GI}$ are fibrant and ${}^{\perp}\mathcal{GI}$ forms the thick class of trivial objects. The situation with $\mathfrak{GF}$ is, perhaps, even more interesting: it induces the abelian model structure where $\mathcal{GF}$ is the class of cofibrant objects and the class $\mathcal{EC}$ of all (Enochs) cotorsion modules provides the fibrant objects. The thick class $\mathcal{T}$ of trivial objects turns out to be the right-hand class of the complete cotorsion pair $(\mathcal{PGF}, \mathcal{T})$ where $\mathcal{PGF}$ denotes the class of all projectively resolved Gorenstein flat modules; in particular, every module in $\mathcal{PGF}$ is Gorenstein projective. On the other hand, each Gorenstein AC-projective module, in the sense of Bravo, Gillespie and Hovey, belongs to $\mathcal{PGF}$.

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