

# Internal sizes in accessible categories

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**Abstract:** The internal size of an object  $M$  inside a given category is, roughly, the least infinite cardinal  $\lambda$  such that any morphism from  $M$  into the colimit of a  $\lambda^+$ -directed system factors through one of the components of the system. The existence spectrum of a category is the class of cardinals  $\lambda$  such that the category has an object of internal size  $\lambda$ . We study the existence spectrum in  $\mu$ -abstract elementary classes ( $\mu$ -AECs), which are, up to equivalence of categories, the same as accessible categories with all morphisms monomorphisms. We show for example that, assuming instances of the singular cardinal hypothesis which follow from a large cardinal axiom,  $\mu$ -AECs which admit intersections have objects of all sufficiently large internal sizes. We also investigate the relationship between internal sizes and cardinalities and analyze a series of examples, including one of Shelah—a certain class of sufficiently-closed constructible models of set theory—which show that the categoricity spectrum can behave very differently depending on whether we look at categoricity in cardinalities or in internal sizes.