

pp Properties of infinite sequences in modules

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Abstract: This talk is a report on part of a long term project in which Philipp Rothmaler and I investigate the model theory of pp definable properties of infinite sequences in modules, and especially concepts of elementary duality for such properties. I will give the general background to this investigation, and then outline a characteristic family of examples.

An interesting class of such properties can be constructed as follows. Let \bar{v} be a finite list of distinct variables (the “type” variables), and let $\bar{x} = \langle x_i \rangle_{i \in I}$ be a possibly infinite list of distinct variables, distinct from \bar{v} (the “parameter” variables). If A is a subset of a left R -module M , $|A| = |I|$, a pp type in the variables \bar{v} over A can be construed as a set of instances of pp formulas in the variables \bar{v} and \bar{x} by a suitable indexing of A . In general, a set $\Phi(\bar{v}, \bar{x})$ of parameter-free (finitary) pp formulas determines properties of sequences $\bar{a} \in M^I$ according to various conditions on the pp type $\Phi(\bar{v}, \bar{a})$: that it be realized in M , that it be finitely satisfiable in M , that it be realized in some elementary extension of M , that it be realized in some extension of M , and so on.

The theory of a left R -module M is determined by the “collapsing” (or not) of pairs of pp formulas (ϕ/ψ) , where $\psi \longrightarrow \phi$ in all left R -modules. We show that for any ring R , some natural classes of modules can be characterized by the “collapsing” of a family of pairs of infinitary properties of the sort just described. We consider injectivity and closely related properties; and flatness and closely related properties; and determine some dualities between these properties.

We are eager to hear suggestions of other interesting properties of infinite sequences in modules worthy of investigation.