Homomorphic images of subdirectly irreducible rings

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Abstract: A (universal) algebra is subdirectly irreducible (SI) if and only if the intersection of all its non-identical congruences is non-identical. By the well-known Birkhoff's theorem, every algebra can be embedded into a product of SI algebras. One can ask, on the other hand, about the characterization of all homomorphic images of SI algebras. Such a characterization was given by Stanovský '01 and independetly by Ježek and Kepka '02 for the variety of grupoids. Analogical results were further obtained also for other varieties: semigroups, lattices, groups, loops, Bol loops, quasigroups and unary algebras. We give a similar characterization for the varieties of rings, radical rings and for their subvarieties of commutative rings. Recall that radical rings (also called quasi-regular rings) are precisely all Jacobson radicals of all rings. They can also be equivalently characterized as such rings that form a group with respect to the adjoint operation $a \circ b = a + b + ab$.