A categorification of the representation theory of finite groups

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Abstract: Many abelian or triangulated categories in algebra, geometry and topology are of the equivariant persuasion: they come in group-indexed families, and are connected by change-of-group functors such as restriction, induction and conjugation. Here we are thinking of categories of things like group representations, equivariant sheaves, G-spectra, G-C*-algebras, and so on. Once decategorified – e.g. by applying K_0 throughout – the resulting algebraic structure is quite well understood, at least in the case of finite groups, and has been axiomatised via Dress's Mackey functors or Bouc's biset functors.

I will explain joint work with Paul Balmer, in which we categorify the notion of Mackey functor in order to understand the underlying richer (2-)categorical information which is also available in all above-mentioned examples. Many such "Mackey 2-functors" arise from additive derivators, but there are others who do not, such as the stable module category or equivariant stable homotopy. In a precise sense, the universal example is provided by a certain bicategory of spans (i.e. correspondences) of finite groupoids, and can be concretely used to prove results. For instance, a computation with such spans shows that the separable monadicity of restriction functors always holds true, which explains previous observations by Paul Balmer, myself and Beren Sanders.