Large parallel volume of compact sets and digitized Boolean models.

Markus Kiderlen

The parallel volume of a set A in n-dimensional Euclidean space (i.e. the volume of the set of all points with distance at most $r > 0$ from A) is a basic tool to define intrinsic volumes, curvature measures, support measures and related notions. Usually parallel volumes for small radii r are considered for this purpose. In the first part of the talk we will discuss recent results concerning the asymptotics of the parallel volume for *large* radii r. These results mainly concern the case where A is *finite* and a polynomial expansion (in r^{-1}) can be obtained. In the more general case of a compact set A, we can only show that up to order 2, the parallel volume of A behaves like the one of the convex hull of A (which is a polynomial) and determine the coefficient of order 3.

In the second part of the talk we discuss applications of parallel volumes to analyze digitizations of a stationary Boolean model Z of random balls. It was shown by R. Miles and other authors in the seventies that the specific intrinsic volumes of Z (mean volume, mean surface area etc. per unit volume) can be expressed by the intensity of the underlying Poisson process and moments of the radii distribution of the typical ball of Z. If the specific intrinsic volumes of Z are estimated from a digitization of Z (i.e. from the knowledge of $Z \cap t\mathbb{Z}^n$, where $t > 0$ is the lattice distance), these formula fail. Based on an early observation of Serra, we show that the results of the first part of the talk can be used to establish asymptotic Miles-type formula for digitized Boolean models complementing recent results of W. Nagel, J. Ohser and K. Schladitz. Asymptotics is understood here for $t \to \infty$, that is, for increasing resolution of the lattice. We give an outlook of how approximate Miles-type formula can even be obtained for a given positive t.

This is joint work with J. Kampf.