NMTP438, topic 5: stationary point process

1. Show that a homogeneous Poisson point process is stationary and isotropic.

2. Based on the interpretation of the Palm distribution determine the Palm distribution and the reduced Palm distribution of a binomial point process.

3. Consider independent random variables U_1 a U_2 with uniform distribution on the interval $[0, a]$, $a > 0$, and the point process Φ in $\mathbb{R}^2$defined as

$$\Phi = \sum_{m,n\in\mathbb{Z}} \delta_{(U_1+ma,U_2+na)}.$$

Determine the Palm distribution and the reduced second-order moment measure of the process. Express its contact distribution function and the nearest-neighbour distribution function.

4. Show that for a homogeneous Poisson process with the intensity λ it holds that $\mathrm{PI} = \mathrm{CE} = 1$, $F(r) = G(r) = 1 - \mathrm{e}^{-\lambda\omega_d r^d}$ and $J(r) = 1$.

5. Determine the pair-correlation function of a binomial point process, provided it exists.

6. Let $Y = \{Y(x) : x \in \mathbb{R}^d\}$ be a weakly stationary Gaussian random field with the mean value μ and the autocovariance function $C(x, y) = \sigma^2 r(x - y)$, where σ^2 denotes the variance and r is the autocorrelation function of the random field Y. Consider the random measure

$$\Psi(B) = \int_B \mathrm{e}^{Y(x)}\,\mathrm{d}x, \quad B \in \mathcal{B}^d.$$

The Cox point process Φ with the driving measure Ψ is called a *log-Gaussian Cox process.* Show that the distribution of Φ is determined by its intensity and its pair-correlation function.

7. Determine the pair-correlation function of

a) the Thomas process,
b) the Matérn cluster process for $d = 2$.

8. For a point process with the hard-core distance $r > 0$ and the intensity λ we define the *coverage density* as $\tau = \lambda|b(o, r/2)|$. It is in fact the mean volume fraction of the union of balls with the centers in the points of the process and the radii $r/2$. Determine the maximum possible value of τ for the following models:

a) Matérn hard-core process type I,
b) Matérn hard-core process type II.