NMTP438, topic 4: binomial, Poisson and Cox point process

1. Show that the mixed binomial point process with the Poisson distribution (with parameter λ) of the number of points N is a Poisson process with the intensity measure $\lambda \frac{\nu(\cdot)}{\nu(B)}$.

2. Let Φ be a Poisson point process with the intensity measure Λ and $B \in \mathcal{B}$ be a given Borel set. Show that $\Phi|_B$ is a Poisson point process and determine its intensity measure.

3. Consider two independent Poisson point processes Φ_1 and Φ_2 with the intensity measures Λ_1 and Λ_2. Show that $\Phi = \Phi_1 + \Phi_2$ is a Poisson process and determine its intensity measure.

4. Let Φ be a Poisson point process with the intensity measure Λ. Determine the covariance $\operatorname{cov}(\Phi(B_1), \Phi(B_2))$ for $B_1, B_2 \in \mathcal{B}$.

5. Let Φ be a binomial point process with n points in B and the measure ν. Determine the covariance $\operatorname{cov}(\Phi(B_1), \Phi(B_2))$ for $B_1, B_2 \in \mathcal{B}$.

6. Let Φ be a mixed Poisson point process with the driving measure $Y \cdot \Lambda$, where Y is a non-negative random variable and Λ is a locally finite diffuse measure. Determine the covariance $\operatorname{cov}(\Phi(B_1), \Phi(B_2))$ for $B_1, B_2 \in \mathcal{B}_0$ and show that it is non-negative.

7. Determine the second-order factorial moment measure of a binomial point process.

8. Determine the Laplace transform of a binomial point process.

9. Dispersion of a random variable $\Phi(B)$ is defined as

$$D(\Phi(B)) = \frac{\operatorname{var} \Phi(B)}{\mathbb{E}\, \Phi(B)}, \ B \in \mathcal{B}_0.$$

Show that

a) for a Poisson process $D(\Phi(B)) = 1$,
b) a binomial process is underdispersed, i.e. $D(\Phi(B)) \leq 1$,
c) a Cox process is overdispersed, i.e. $D(\Phi(B)) \geq 1$.

10. Let Y be a random variable with a gamma distribution. Show that the corresponding mixed Poisson process Φ is a negative binomial process, i.e. that $\Phi(B)$ has a negative binomial distribution for every $B \in \mathcal{B}_0$.