(1) $\int_0^1 x^{ax}\,dx$ KONV. VŽDY $\Leftarrow$ omezenost $f(x) = x^{ax}$ (rozšíříme $(0,1]$, $f(x) \to 1$, $x \to 0+$).

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(2) $\int_0^2 \frac{|\ln x|^\lambda}{\sqrt{1-x}}\,dx$ KONV. $\Leftrightarrow$ $\boxed{\lambda > -\frac{1}{2}}$

(i) $f(x) \sim (1-x)^{\lambda-\frac{1}{2}}$, $x \to 1-$ ... $\lambda - \frac{1}{2} > -1$

neboť $|\ln x| \sim |x-1|$, $x \to 1$.

(ii) $f(x) \le \frac{1}{\sqrt{x}}$, x blízko $0+$; neboť $\sqrt{x} f(x) \to 0$.

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(3) $\int_1^{+\infty} \sin(x^\alpha)\,dx$ KONV. $\Leftrightarrow$ $\alpha > 1$ nebo $\alpha < -1$

(i) $\alpha > 0$: subst. $x^\alpha = y \rightsquigarrow \int_1^{+\infty} \frac{\sin y}{\alpha y^{1-1/\alpha}}\,dy$ ... $\boxed{1-1/\alpha > 0}$ .. Dirichlet

(ii) $\alpha < 0$: $f(x) \sim x^\alpha$, $x \to +\infty$ ... $\boxed{\alpha < -1}$

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(4) $\int_0^{+\infty} x \cos x^4\,dx$ = KONV. subst. $x^4 = y \rightsquigarrow \int_0^{+\infty} \frac{\cos y}{4 y^{3/2}}\,dy$

(i) $y \in (0,1]$: $|f(y)| \le \frac{1}{\sqrt{y}}$

(ii) $y \in [1,+\infty)$: Dirichlet