(i) $\cos\frac{1}{10}$ ... chyba $< 10^{-3}$.

víme: $$\cos x = \underbrace{1-\frac{x^2}{2!}+\frac{x^4}{4!}+\dots+(-1)^m\frac{x^{2m}}{(2m)!}+0\cdot x^{2m+1}}_{T^{\cos}_{2m+1}(x)} + R_{2m+2}(x)$$

Lagrange: $$|R_{2m+2}(x)| \le \frac{|x|^{2m+2}}{(2m+2)!}\underbrace{|\cos^{(2m+2)}(\xi)|}_{\le 1} \le \frac{|x|^{2m+2}}{(2m+2)!}$$

$m=0$: $\cos\frac{1}{10} = 1 + R_2\left(\frac{1}{10}\right)$; $\quad \left|R_2\left(\frac{1}{10}\right)\right| \le \frac{1}{2}\left(\frac{1}{10}\right)^2 = 5\cdot 10^{-3}$

$m=1$: $\cos\frac{1}{10} = \underbrace{1-\frac{1}{2}\left(\frac{1}{10}\right)^2}_{0.995} + R_4\left(\frac{1}{10}\right)$; $\quad \left|R_4\left(\frac{1}{10}\right)\right| \le \frac{1}{4!}\left(\frac{1}{10}\right)^4$

$\doteq 4{,}17\cdot 10^{-6}$ OK

(ii) $$\sqrt[5]{250} = (\underbrace{243}_{3^5}+7)^{1/5} = 3\left(1+\frac{7}{243}\right)^{1/5}.$$

$$(1+x)^\alpha = \sum_{k=0}^{m}\binom{\alpha}{k}x^k + R_{m+1}(x); \quad |R_{m+1}(x)| \le \frac{|x|^{m+1}}{m+1}$$

$\forall x>0,\ \alpha\in(0,1)$.

$m=1$: $\sqrt[5]{250} = 3\left(1+\frac{1}{5}\cdot\frac{7}{243} + R_2\left(\frac{7}{243}\right)\right)$

kde $3\cdot R_2\left(\frac{7}{243}\right) \le 3\cdot\left(\frac{7}{243}\right)^2\cdot\frac{1}{2} = 0.0012\dots$

$m=2$: $\sqrt[5]{250} = 3\left(1+\frac{1}{5}\cdot\frac{7}{243} + \frac{1}{5}\left(-\frac{4}{5}\right)\cdot\frac{1}{2}\left(\frac{7}{243}\right)^2 + R_3\left(\frac{7}{243}\right)\right)$

kde $3\cdot R_3\left(\frac{7}{243}\right) \le 3\cdot\left(\frac{7}{243}\right)^3\cdot\frac{1}{3} \doteq 2{,}39\cdot 10^{-5}$.. OK

(iii) $\log_{10} 11 = ???$

(iv) $\arcsin(0{,}45)$ s chybou 10^{-3}

víme: $$\arcsin x = \sum_{k=0}^{\infty} \frac{(2k-1)!!}{(2k)!!}\,\frac{x^{2k+1}}{2k+1} = \underbrace{\sum_{k=0}^{n} \cdots}_{T_{2n+2}^{\arcsin}(x)} + \underbrace{\sum_{k=n+1}^{\infty} \cdots}_{R_{2n+3}(x)}$$

$\forall x \in (-1,1)$

odhad zbytku: víme:

$$|R_{2n+3}(x)| \le \sum_{k=n+1}^{\infty} \frac{x^{2k+1}}{2k+1} \le \frac{x^{2n+3}}{2n+3}\left(1+x^2+\cdots\right)$$

$$= \frac{x^{2n+3}}{2n+3}\cdot\frac{1}{1-x^2}.$$

$x = 0.45 < \frac{1}{2}$

$x^2 < \frac{1}{4}. \implies \frac{1}{1-x^2} \le \frac{1}{1-1/4} = \frac{4}{3}.$

$n=1: \quad R_5(0{,}45) \le \frac{1}{5}(0{,}45)^5\cdot\frac{4}{3} = 4{,}9\cdot 10^{-3}$ (ne)

$n=2: \quad R_7(0{,}45) \le \frac{1}{7}(0{,}45)^7\cdot\frac{4}{3} \doteq 7{,}11\cdot 10^{-4}$ OK.

(*) $$S(x) = \sum_{k=1}^{\infty} (-1)^k \frac{k x^k}{(2k-1)!} = x\left(\sum_{k=1}^{\infty} (-1)^k \frac{k x^{k-1}}{(2k-1)!}\right) = x\cdot\left(\sum_{k=1}^{\infty} (-1)^k \frac{x^k}{(2k-1)!}\right)'$$

víme: $$\sin x = \sum_{l=1}^{\infty} (-1)^{l-1} \frac{x^{2l-1}}{(2l-1)!}$$

$$-x\cdot\sin x = \sum_{l=1}^{\infty} (-1)^{l} \frac{x^{2l}}{(2l-1)!}$$

$\implies$

$$S(x) = x\left(-\sqrt{x}\,\sin\sqrt{x}\right)'$$

$\forall x > 0.$