(6a) $\phi(y) = \int_0^{\pi} (y')^3 \, dx$; $y(0) = 0$, $y(\pi) = a\pi$; $a \neq 0$ konst.

$f = r^3$;
$f_y = 0$
$f_r = 3r^2$

(E.L.) $-[3(y')^2]' = 0$

$(y')^2 \equiv C_1 \; (\geq 0)$

$\left.\begin{array}{l} y' \equiv \pm\sqrt{C_2} \\ y' \text{ - spojité} \end{array}\right\} \Rightarrow y' \equiv c \; ; \; c \in \mathbb{R}.$

$y = cx + d$ ~ $\boxed{y = ax}$ extremála.

$f_{rr} = 6r$
$f_{yr} = 0$
$f_{yy} = 0$

$P(x) = 6a^2 > 0$ (J) $(6a^2u')' = 0$
$Q(x) = 0$

$u'' = 0$

$\boxed{u = \alpha x + \beta}$

$\left.\begin{array}{l} u(0) = 0 \text{ ; netriviální} \\ u(x) = \alpha x ; \; \alpha \neq 0 \end{array}\right\} \Rightarrow \nexists$ konjugovaný bod

$\Rightarrow$ $\boxed{y = ax \text{ je lok. min.}}$

(6b) $\phi(y) = \int_0^1 (y')^3 + (3y')^2 + y' \, dx$; $y(0) = 0$, $y(1) = 1$

$f = r^3 + 3r^2 + r$
$f_r = 3r^2 + 6r + 1$
$f_y = 0$.

(E.L) $-\left(3(y')^2 + 6y' + 1\right)' = 0$

$3(y')^2 + 6y' = C$

$\Rightarrow y' = C_1$

$y = C_1 x + C_2$

extremála:

$\boxed{y = x}$

$f_{rr} = 6r + 6$ $\qquad P(1) = 6\cdot 1 + 6 = 12 > 0$

$f_{yr} = 0 = f_{yy}$ $\qquad Q(1) = 0$

(J) $\qquad (12u')' = 0$

$u'' = 0$

$u = \alpha x + \beta$ .. $\nexists$ konj. bod. $\boxed{\Rightarrow y = x \text{ je lok. min}}$

(6c) $\phi(y) = \int_1^2 x^2 (y')^3 dx$; $\qquad y(1) = 0$, $y(2) = \ln 2$

$f = x^2 r^3$ $\qquad$ (E.L.) $\quad (-3x^2(y')^2)' = 0$

$f_r = 3x^2 r^2$ $\qquad x^2 (y')^2 = C$

$f_y = 0$ $\qquad y' = \frac{\pm C}{x} = \frac{d}{x}$

$y = d \ln|x| + \tilde{d}$.

$\boxed{y = \ln x}$ extremála

$f_{rr} = 6x^2 r$ $\qquad P(x) = 6\cdot x^2 \cdot \frac{1}{x} = 6x > 0$

$f_{yr} = f_{yy} = 0$ $\qquad Q(x) = 0$

(J) $\qquad (6xu')' = 0$

$xu' = c$

$u' = \frac{c}{x} \rightarrow u = c\ln x + \tilde{c}$

$\nexists$ konj. bod.

$\Rightarrow \boxed{y = \ln x}$ je lok. minimum

6d $\quad \phi(y) = \int_1^2 y^3 (y')^3 \, dx \; ; \quad y(1) = 2 \; ; \; y(2) = 2\sqrt{2}.$

$f = y^3 r^3$

$f_r = 3 y^3 r^2$

$f_y = 3 y^2 r^3$

(E.L.) $\quad -\left[3y^3 (y')^2\right]' + 3 y^2 (y')^3 = 0$

$\left(-y^3 (y')^2\right)' + y^2 (y')^3 = 0$

$-3y^2 (y')^3 - 2y^3 (y') y'' + y^2 (y')^3 = 0$

$y^2 (y')^3 + y^3 y' y'' = 0$

$y^3 (y')^2 \left[\frac{y'}{y} + \frac{y''}{y'}\right] = 0$

$\boxed{y \equiv 0}$

$(y')^2 \equiv 0 \Rightarrow \boxed{y \equiv C}$

Obecně je třeba uvažovat i napojení těchto řešení; $\nexists$ ne podmínky C^1

3. rce: $\quad \frac{y'}{y} + \frac{y''}{y'} = 0$

$\ln y + \ln y' = c$

$2y y' = 2d$

$y^2 = 2dx + \tilde{d}$

$\boxed{y = \sqrt{2dx + \tilde{d}}}$

okr. podm. $\Rightarrow$

$\boxed{y = \sqrt{4x}}$ je extremála

$y' = (2\sqrt{x})' = \frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$

$f_{rr} = 6 y^3 r$

$f_{yr} = 9 y^2 r^2$

$f_{yy} = 6 y r^3$

$P = 6 y^3 r \Big|_{\substack{y = 2x^{\frac{1}{2}} \\ r = x^{-\frac{1}{2}}}} = 6 \left(2x^{\frac{1}{2}}\right)^3 x^{-\frac{1}{2}} = 48 x > 0$

$Q = 6 y r^3 - \underbrace{(9 y^2 r^2)'}_{=0} = 6 \cdot 2x^{\frac{1}{2}} \cdot x^{-3/2} = \frac{12}{x}$

(J) $\left(48x u'\right)' - \frac{12}{x} u = 0$

$$4xu'' + 4u' - \frac{u}{x} = 0 \quad ; \ x$$

$$4x^2u'' + 4xu' - u = 0$$ Eulerova rce...

$$\boxed{u(t) = x^\lambda}$$

$$4\lambda(\lambda-1) + 4\lambda - 1 = 0$$

$$4\lambda^2 - 1 = 0$$

$$\lambda = \pm \frac{1}{2}$$

$$F.S. = \left\{ x^{\frac{1}{2}}, x^{-\frac{1}{2}} \right\}$$

$$u(t) = C_1 x^{\frac{1}{2}} + C_2 x^{-\frac{1}{2}}$$

$$= x^{-\frac{1}{2}}\left(C_1 x + C_2\right)$$

obecné řešení nemá nejvýše 1 nulový bod $\Rightarrow \nexists$ konj. bod netriviální v $[1,2]$

$$\boxed{\Rightarrow y = 2\sqrt{x} \text{ je lokální minimum}}$$