2.1 $f = xyz$

$\Gamma = \{ x^2+y^2+z^2 = 1 \}$

$g = x^2+y^2+z^2-1$

$\nabla g = (2x, 2y, 2z) \neq \underline{0}$ na Γ.

$\nabla f = \lambda \nabla g$

$yz = 2\lambda x$ $\quad$ $x=0$. 1.rce $\Rightarrow \boxed{y=0} \vee z=0$

$xz = 2\lambda y$ $\quad$ $x=y=0$

$xy = 2\lambda z$ $\quad$ $z\neq 0$: 3.rce $\Rightarrow \lambda = 0$.

$\boxed{z=\pm 1}$

$(0,0,\pm 1);\ (0,\pm 1,0);\ (\pm 1,0,0)$. $\quad$ $f=0$.

$\left.\begin{array}{l} x\neq 0 \\ y\neq 0 \\ z\neq 0 \end{array}\right\}$ $\quad \dfrac{1.\text{rce}}{2.\text{rce}}: \quad \dfrac{y}{x} = \dfrac{x}{y}$

$y^2 = x^2 \Rightarrow |x| = |y|$

$\dfrac{1.\text{rce}}{3.\text{rce}}: \quad x^2 = z^2 \quad |x| = |z|$.

$(x,y,z) = \left(\pm \frac{1}{\sqrt{3}}, \pm \frac{1}{\sqrt{3}}, \pm \frac{1}{\sqrt{3}} \right)$.

$f = \pm \dfrac{1}{3\sqrt{3}}$.

2.2. $f = \sin x \sin y \sin z$

$$M = \{x+y+z = \tfrac{\pi}{2}\} \cap \{x, y, z > 0\}.$$

$$g = x+y+z-\tfrac{\pi}{2}$$

(i) $\nabla g = 0$: $\emptyset$

(ii) $\nabla f = \lambda \nabla g$

$$\cos x \sin y \sin z = \lambda$$
$$\sin x \cos y \sin z = \lambda$$
$$\sin x \sin y \cos z = \lambda$$

$x, y, z \in (0, \frac{\pi}{2}) \Rightarrow$ všechno je $\neq 0$.

1. & 2. rce: $\cos x \sin y \cancel{\sin z} = \sin x \cos y \cancel{\sin z}$

$$\operatorname{tg} y = \operatorname{tg} x$$

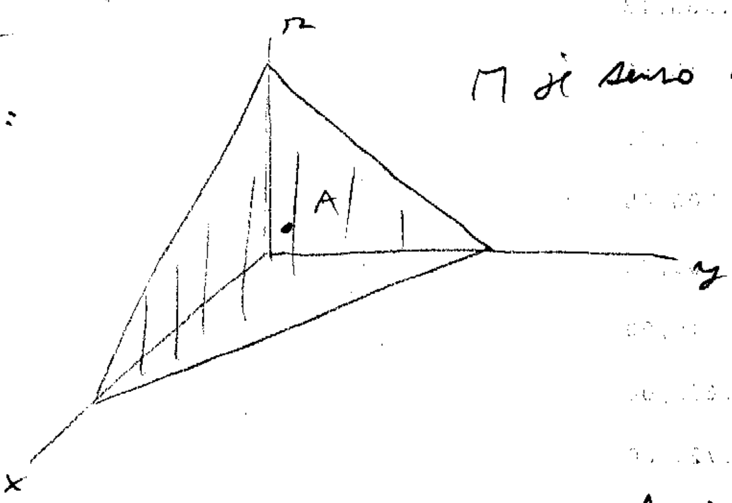
$\Rightarrow x = y$ (tg prostá na $(0, \frac{\pi}{2})$)

analogicky: $x = y = z = \frac{\pi}{6}$.

$A = \left(\frac{\pi}{6}, \frac{\pi}{6}, \frac{\pi}{6}\right)$. ← podezřelý bod:

$f(A) = \left(\sin \frac{\pi}{6}\right)^3 = \frac{1}{8}$.

diskuse:



M je šikmý Δ bez okraje.

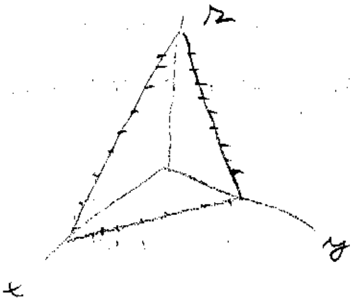
$f > 0$ na M; $f \to 0$ blízko okraje

$\Rightarrow$ A je glob. max.

min. $\nexists$

2.3. $f = xy^2z^3$; $M = \{g=0\} \cap \{x,y,z>0\}$

$$g = x + 2y + 3z - a \; ; \quad a>0 \text{ pevné.}$$



podezřelé body:

(i) f, g nehladké $\emptyset$

(ii) „okraj" plochy ... $\emptyset$
(není součástí)

(iii) $\nabla g = 0$: $\nabla g = (1,2,3) \neq 0$ všude ... $\emptyset$

(iv) $\nabla f = \lambda \nabla g$

$$y^2z^3 = \lambda$$
$$2xyz^3 = 2\lambda$$
$$3xy^2z^2 = 3\lambda$$

zjevně $\lambda \neq 0$ (neboť $x,y,z>0$).

1. rce / 2. rce $\rightarrow$ $x = y$

1. rce / 3. rce $\rightarrow$ $z = x$;

$x + 2x + 3x = a$; $x = y = z = \frac{1}{6}a$;

$A = \left(\frac{a}{6}, \frac{a}{6}, \frac{a}{6}\right)$ ... jediný podezřelý bod.

Diskuse: $f>0$ na M

$f \rightarrow 0$ blízko okraje

$\Rightarrow$ A ... glob. MAX

min $\nexists$

2.4. $f = x + y$

$M = \{ x^3 + y^3 - 2xy = 0 \} \cap \{ x, y \geq 0 \}$.

uzavřené; omezené: $x = r \cdot \cos\varphi$ 1. kvadrant $\varphi \in [0, \frac{\pi}{2}]$

$y = r \sin\varphi$

$$r^3 (\cos^3\varphi + \sin^3\varphi) = r^2\, 2 \cos\varphi \sin\varphi$$

$$r = \frac{\sin 2\varphi}{\cos^3\varphi + \sin^3\varphi} \leq \frac{1}{\left(\frac{1}{\sqrt{2}}\right)^3};$$

neboť: $\cos^2\varphi + \sin^2\varphi = 1 \Rightarrow \cos^2\varphi \geq \frac{1}{2}$ nebo $\sin^2\varphi \geq \frac{1}{2}$.

---

podezřelé body:

(i) krajní: $x = 0$ nebo $y = 0$: $(0,0)$: $f = 0$.

(ii) $\nabla g = 0$

$$3x^2 - 2y = 0$$
$$3y^2 - 2x = 0\,;$$

$(0,0)$ nebo $x \neq 0, y \neq 0$: podělím $\frac{x^2}{y^2} = \frac{y}{x}$

$x^3 = y^3$

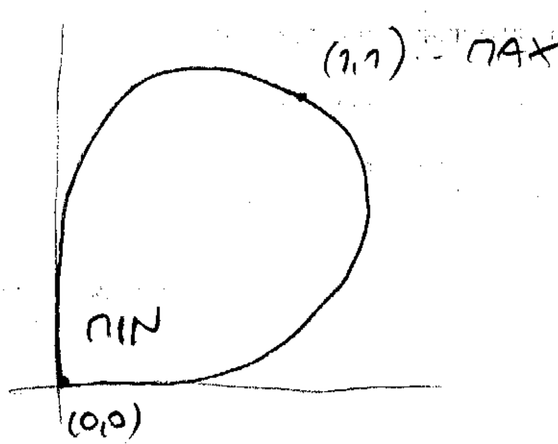
$(1,1)$: $f = 2$. $\leftarrow$ $x = y$

(iii) $\nabla f = \lambda \nabla g$

$$1 = \lambda\,(3x^2 - 2y)$$
$$1 = \lambda\,(3y^2 - 2x)$$
$$0 = \lambda\left(3(x^2 - y^2) + 2(x - y)\right).$$

zjevně $\lambda \neq 0$; $x = y$ již máme; zbývá

$0 = (3(x+y) + 2)$ ... $\emptyset$ pro $x, y \geq 0$.

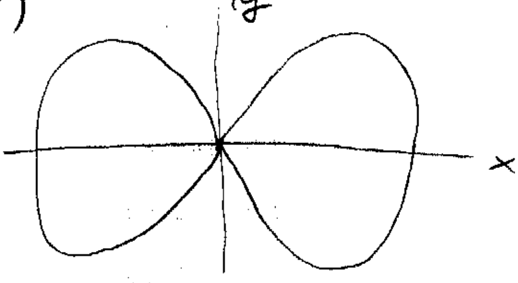


2.5. $f = x$; $M = \{ (x^2+y^2)^2 - 2(x^2-y^2) = 0 \}$.

M uzavřené ; omezené: $x = r\cos\varphi$
$y = r\sin\varphi$

$(r^2)^2 = 2r^2(\cos^2\varphi - \sin^2\varphi)$

$r^2 = 2\cos 2\varphi \leq 2$.



podezřelé body:

(i) $\nabla g = 0$:
$$2(x^2+y^2)\cdot 2x - 4x = 0$$
$$2(x^2+y^2)\cdot 2y + 4y = 0.$$

1. rce: (α) $x = 0$; $\to y = 0$ (z rovnice pro M)

$(0,0)$ .. $f = 0$

(β) $x \neq 0$: $x^2+y^2 = 1$; 2. rce $\to$ $8y = 0$

$y = 0$; $\to$ $x = \cancel{0}, \pm 1$

$A = (\pm 1, 0)$; $f(A) = \pm 1$.

(ii) $\nabla f = \lambda \nabla g$
$$1 = \lambda [4x(x^2+y^2-1)]$$
$$0 = \lambda [4y(x^2+y^2+1)].$$

2. rce: (α) $\lambda = 0$: spor s 1. rcí

(β) $y = 0$: již máme

celkem: $(-1,0)$: MIN

$(1,0)$: MAX.

2.6. $f = x^2 + y$; $M = \{y^3 - 4y + x^2 = 0\} \cap \{y \geq 0\}$.

M uzavřené ; omezené :

$$y^3 - 4y = -x^2 \leq 0$$
$$y(y^2 - 2) \leq 0$$
$$y^2 - 2 \leq 0 \;:\; y^2 \leq 2 \;;\; \underline{y \leq \sqrt{2}}$$

$$x^2 = 4y - y^3 \leq 4y$$
$$|x| \leq 2\sqrt{\sqrt{2}}.$$

---

podezřelé body

(i) krajní : $y = 0$: $x = 0$: $(0,0)$; $f = 0$

(ii) $\nabla g = 0$: $2x = 0$: $x = 0$

$3y^2 - 4 = 0$: $y = \pm\frac{2}{\sqrt{3}}$ $\notin M$

(iii) $\nabla f = \lambda \nabla g$: $2x = \lambda 2x$

$1 = \lambda(3y^2 - 4)$

1. rce : (α) $x = 0$; $\to$ $y^3 - 4y = 0$ (rce M)

$y(y^2 - 4) = 0$

$y = 0$ (máme) nebo $y = 2$:

$(0,2)$ .. $f = 2$.

(β) $x \neq 0$: $\lambda = 1$; 2. rce $3y^2 - 4 = 1$

$y = \sqrt{\frac{5}{3}}$

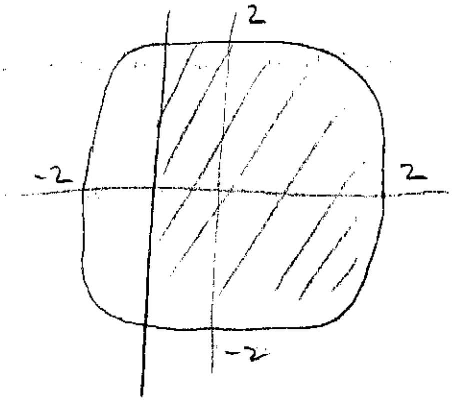
rce pro M : $\sqrt{\frac{5}{3}}\Big(\underbrace{\frac{5}{3} - 4}_{<0}\Big) + x^2 = 0$ $\to$ nemá řešení!

Závěr : $(0,0)$ ... MIN

$(0,2)$ .. MAX.

2.7. $f = x^4 y$; $M = \{x^4+y^4 \le 16\} \cap \{x \ge -1\}$

M omezená ($|x|, |y| \le 2$)
uzavřená.



podezřelé body :

(i) uvnitř $\nabla f = 0$: $4x^3y = 0$
$x^4 = 0$

$x = 0$; $y \in (-2, 2)$: $f = 0$.

(ii) vůči $x = -1$: $f = y$; $y \in [-\sqrt[4]{15}, \sqrt[4]{15}]$.

$\Rightarrow$ $(-1, -\sqrt[4]{15})$ $f = -\sqrt[4]{15} \doteq -1.97$
$(-1, \sqrt[4]{15})$ $f \doteq 1.97$

(iii) vůči $\{x^4+y^4 = 16\} = \Gamma$

(α) $\nabla g = 0$: $4x^3 = 0$, $4y^3 = 0$ $(x,y) = (0,0) \notin \Gamma$

(β) $\nabla f = \lambda \nabla g$ $4x^3y = \lambda 4x^3$
$x^4 = \lambda 4y^3$

1. rce: (i) $x = 0$ — $y = \pm 2$; $f = 0$ (již máme)

(ii) $x \ne 0$: $y = \lambda$; 2. rce: $x^4 = 4y^4$;
$x = \pm\sqrt{2}\, y$

rce pro Γ: $x^4 + \frac{1}{4}x^4 = 16$

$x^4 = \frac{4 \cdot 16}{5}$; $x = \pm\frac{2\sqrt{2}}{\sqrt[4]{5}} \doteq \pm 1.89$

body $\left(\pm\frac{2\sqrt{2}}{\sqrt[4]{5}}, \pm\frac{2}{\sqrt[4]{5}}\right)$;

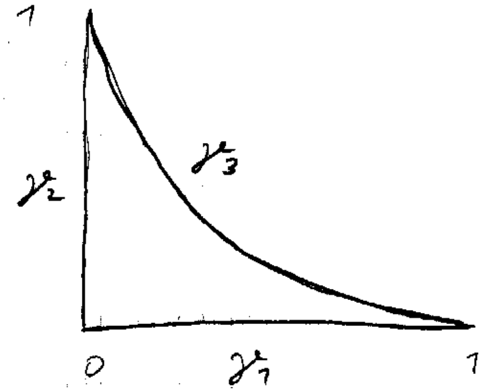
Γ můžeme $\left(+\frac{2\sqrt{2}}{\sqrt[4]{5}}, \pm\frac{2}{\sqrt[4]{5}}\right)$; $f = \pm\frac{128}{5\sqrt[4]{5}} = \pm 11.45$

MAX | MIN

2.8. $f = 2x + 4y$; $M = \{x^{1/4} + y^{1/4} \le 1\} \cap \{x, y \ge 0\}$

omezená ($x, y \in [0,1]$); uzavřené.

podezřelé body:



(1) uvnitř: $\nabla f = 0$ … nikdy

(2) hranice + rohy

$(0,0)$ $f = 0$

$(0,1)$ $f = 2$

$(1,0)$ $f = 4$

(3) úseky hranice: podél γ_1, γ_2 f roste

$$\left(\frac{\partial f}{\partial x} = 2, \ \frac{\partial f}{\partial y} = 4\right)$$

(4) $\gamma_3 = \{x^{1/4} + y^{1/4} - 1\} \cap \{x, y > 0\} = M$

$\nabla g = 0$: $\frac{1}{4} x^{-3/4} = 0$ … nikdy

$$\nabla f = \lambda \nabla g: \quad 2 = \lambda \cdot \frac{1}{4} x^{-3/4}$$
$$4 = \lambda \cdot \frac{1}{4} y^{-3/4}$$

nutně $\lambda \neq 0$: 2. rce / 1. rce $\quad 2 = \left(\frac{x}{y}\right)^{3/4}$;

$$x = 2^{4/3} y$$

rce pro M: $(2^{1/3} + 1) y^{1/4} = 1$;

odtud $C = \left(2^{4/3} (2^{1/3}+1)^{-4}, (2^{1/3}+1)^{-4}\right) \doteq (0.10, 0.04)$

$f(C) \doteq 0.36$

závěr: $(0,0)$ – MIN

$(1,0)$ – MAX

2.9. $f = e^{-r^2}(x^2+xy+y^2)$

$M = \{x^2+y^2=1\} \cap \{|r| \le 1\}$.

Pozorování: $f = e^{-r^2}(1+xy)$ na M;

dále: $1+xy \ge \frac{1}{2}$; neboť $2xy \le x^2+y^2=1$

---

stačí vyšetřit separátně

$\varphi(r) = e^{-r^2}$ na $[-1,1]$ ... $r=0$ max

$r = \pm 1$ min

$\psi(x,y) = xy$ na $M = \{x^2+y^2=1\}$.

podezřelé body:

(i) $\nabla g = 0$: $\nabla g = (2x, 2y) \neq 0$ na M

(ii) $\nabla\psi = \lambda \nabla g$ $\quad y = \lambda \cdot 2x$

$x = \lambda 2y$

musí $\lambda \neq 0$ (jinak $x=y=0 \notin M$)

a tedy $x, y \neq 0$.

$y/x = \frac{2x}{2y}$; $y^2 = x^2$: $\left(\pm\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{2}}\right)$.

$y = \pm x$.

$\psi = \pm \frac{1}{2}$.

A

Celkem: MAX: $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$

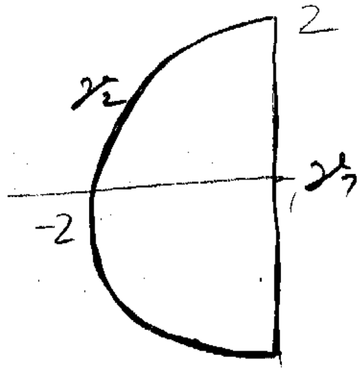
$\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right)$

MIN: $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, \pm 1\right)$

$\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \pm 1\right)$

2.10. $f = -y^2 + x^2 + \frac{4}{3}x^3$;

$M = \{x^2+y^2 \le 4\} \cap \{x \le 0\}$



M -- kompaktní zjevně.

podezřelé body

(1) vnitřek $\nabla f = 0$: $2x + 4x^2 = 0$: $2x(1+2x)=0$

$-2y = 0$

$(-\frac{1}{2}, 0)$: $f \doteq 0.42$

(2) rohy : $(0,\pm 2)$: $f = -4$

(3) úsek : γ_1 : $\varphi(t) = f(0,t) = -t^2$; $t \in [-2,2]$.

$t=0$: $(0,0)$ -- lok. max : $f = 0$.

(4) úsek $\gamma_2 = \{x^2+y^2=4\} \cap \{x<0\}$

(i) $\nabla g = 0$ nikde ; $g = (2x, 2y)$

(ii) $\nabla f = \lambda \nabla g$ $\quad 2x(1+2x) = \lambda 2x$

$-2y = \lambda 2y$

jistě $x \neq 0$; 1. rce $\to$ $1+2x = \lambda$

2. rce : $-2y = (1+2x)2y$

(α) $y=0$: $(-2,0)$ $f \doteq -6.67$

(β) $y \neq 0$: $-1 = 1+2x$

$x = -1$; $y = \pm\sqrt{3}$.

$f(-1,\pm\sqrt{3}) = 3+1-\frac{4}{3} = 2.67$

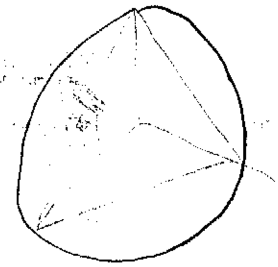
Závěr $(-1,\pm\sqrt{3})$ -- max

$(0,\pm 2)$ -- min

3.11. $f = xy + yz$

$M = \{x^2+y^2+z^2=1\} \cap \{x+y+z=1\}$

kružnice   sféra   roviny



$[0,0,1]$

$g_1 = x^2+y^2+z^2-1$

$g_2 = x+y+z-1$

$$\frac{\partial g_{1,2}}{\partial(x,y,z)} = \begin{pmatrix} 2x, & 2y, & 2z \\ 1, & 1, & 1 \end{pmatrix}$$

LN: $x = y = z$

$g_1 = 0: \; x = y = z = \frac{1}{3}$

$g_2 = 3\cdot\left(\frac{1}{3}\right)^2 \neq 1$

$\exists \lambda, \mu \in \mathbb{R}$

$\nabla f = \lambda \nabla g_1 + \mu \nabla g_2$

$\nabla f = (y, x+z, y)$

$y = \lambda \cdot 2x + \mu$

$x+z = \lambda 2y + \mu$

$y = \lambda 2z + \mu$

1. & 3. rce: $2\lambda x = 2\lambda y$

$\lambda(x-y) = 0$

(α) $\lambda = 0$: 2 rce: $x + z = y$

$\boxed{\begin{matrix} y = \mu \\ x+z = \mu \end{matrix}}$

g_1: $2y = 1$, $\boxed{y = \frac{1}{2}}$

$x + z = \frac{1}{2}$; $z = (\frac{1}{2} - x)$

$x^2 + z^2 = 1 - y^2 = 3/4$

$x^2 + (x - \frac{1}{2})^2 = 3/4$

$2x^2 - x + \frac{1}{4} = 3/4$

$2x^2 - x - \frac{1}{2} = 0 \quad |\cdot 2$

$4x^2 - 2x - 1 = 0 \qquad D = 4 + 16 = 20$

$x_{1,2} = \frac{2 \pm 2\sqrt{5}}{8}$

pokr. 2.11. $x = \frac{1 \pm \sqrt{5}}{4}$; $z = \frac{+1 \mp \sqrt{5}}{4}$.

podezřelé body: $\left(\frac{1+\sqrt{5}}{4}, \frac{1}{2}, \frac{1-\sqrt{5}}{4}\right)$ $\boxed{f = \frac{1}{4}}$

$\left(\frac{1-\sqrt{5}}{4}, \frac{1}{2}, \frac{1+\sqrt{5}}{4}\right)$. $\boxed{f = \frac{1}{4}}$

zk.: $\frac{(1 \pm \sqrt{5})^2}{4^2} = \frac{1 \pm 2\sqrt{5} + 5}{16} = 2\frac{6}{16} + \frac{1}{4} = \frac{12+4}{16} = 1.$

$f = y(x+z) \ldots \underline{\underline{\frac{1}{4}}}.$

(B): $x = y$: $2x + z = 1$; $z = 1 - 2x$

$$2x^2 + z^2 = 1$$

$$2x^2 + (1-2x)^2 = 1$$

$$2x^2 + \cancel{1} - 4x + 4x^2 = \cancel{1}$$

$$6x^2 - 4x = 0$$

$$3x^2 - 2x = 0$$

$x(3x-2) = 0$; $x = 0$

$x = 2/3$

podezřelé body: $(0, 0, 1)$ 0 min.

$\left(\frac{2}{3}, \frac{2}{3}, -\frac{1}{3}\right)$ $\frac{2}{3}\left(\frac{1}{3}\right) = \frac{2}{9} \doteq 0.22$

zk.: $\frac{4}{9} + \frac{4}{9} + \frac{1}{9} = \frac{9}{9}$. ok

Závěr: $(0,0,1)$ $f = 0$ MIN

2 body nahoře $f = \frac{1}{4}$ MAX

$\underline{2.12.}$ $f = (x^2+7y^2)\, e^{-(2x^2+y^2)}$

$M = \{ x^2+4y^2 \leq 1 \}$.

om. & uzavř.: $\Rightarrow \exists$ glob. extr.

$\nabla f = \underset{\sim}{0}$: $\frac{\partial f}{\partial x} = e^{-(2x^2+y^2)} \left[2x - 4x(x^2+7y^2) \right]$

$\frac{\partial f}{\partial y} = e^{-(2x^2+y^2)} \left[14y - 2y(x^2+7y^2) \right]$

$0 = x\left(2-4(x^2+7y^2)\right)$

$0 = y\left(14-2(x^2+7y^2)\right)$

1. rce: (a) $x \neq 0$; musí $\left(2-4(x^2+7y^2)\right) = 0$;

jak (2. rce) $\rightarrow y=0$

(b) $x=0$; $14-14y^2=0$

$y=0$ nebo $y=\pm 1$ $\quad (0,\pm 1) \notin M$.

$y=0$; $2-4x^2=0$

$x^2 = \frac{1}{2}$

$x = \pm \frac{1}{\sqrt{2}}$

$(0,0)$	$f=0$	0
$(\pm\frac{1}{\sqrt{2}},0)$	$\frac{1}{2}e^{-1}$	0.18

hranice: $2x\left(1-2(x^2+7y^2)\right) = \lambda \cdot 2x$

$2y(7-\underbrace{(x^2+7y^2)}_{C}) = \lambda 8y$

$x=0$; $y=\pm\frac{1}{2}$ $\qquad f = \frac{7}{4}\cdot e^{-\frac{1}{4}}$ $\quad 1.36$

$y=0$; $x=\pm 1$. $\qquad f = e^{-2}$ $\quad 0.14$

$x,y \neq 0$: $\quad \frac{1-2C}{7-C} = \frac{1}{4}$

$4-8C = 7-C$

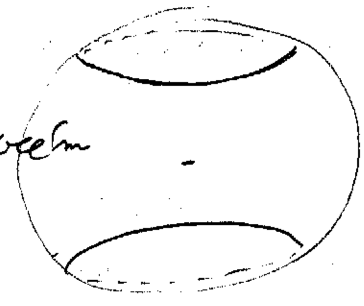
$-3 = 7C$

n.ř.: $C \geq 0$.

2.13. $f = z + e^{xy}$; $M = \underbrace{\{x^2+y^2+z^2=1\}}_{\text{sféra}} \cap \underbrace{\{x^2+y^2=z^2\}}_{\text{kužel}}$

$g_1 = x^2+y^2+z^2-1$

$g_2 = x^2+y^2-z^2$



Pozn.: lze zjednodušit na problém v (x,y); neboť $z = \pm\frac{1}{\sqrt{2}}$ na M.

M omezená, uzavřená.

podezřelé body:

(1) $h\begin{pmatrix}\nabla g_1 \\ \nabla g_2\end{pmatrix} < 2$: $\begin{pmatrix} 2x, 2y, 2z \\ 2x, 2y, -2z \end{pmatrix}$

$-xz = 0$ & $yz = 0$: $\rightarrow$ $z = 0$ (spor s M)

nebo $x = y = 0$ (spor s M)

(2) $\nabla f = \lambda_1 \nabla g_1 + \lambda_2 \nabla g_2$

$$y e^{xy} = \lambda_1 2x + \lambda_2 2x$$
$$x e^{xy} = \lambda_1 2y + \lambda_2 2y$$
$$1 = \lambda_1 2z + \lambda_2 (-2z)$$

1. rce − 2. rce: $(y-x)e^{xy} = (x-y)\,2\,(\lambda_1+\lambda_2)$

(a) $x = y$: (víme: $z^2 = 1/2$), odkud

$2x^2 = \frac{1}{2}$; $x = \pm\frac{1}{4}$;

body: $\left(\frac{1}{4}, \frac{1}{4}, \pm\frac{1}{\sqrt{2}}\right)$

$\left(-\frac{1}{4}, -\frac{1}{4}, \pm\frac{1}{\sqrt{2}}\right)$;

zřejmě: MAX pro $z = \frac{1}{\sqrt{2}}$,

MIN pro $z = -\frac{1}{\sqrt{2}}$.

2.13 – pokr.:

(b) $x \neq y$: $\quad e^{xy} = -2(\lambda_1+\lambda_2) \neq 0$

1. rce: $\quad -2(\lambda_1+\lambda_2)x = 2x(\lambda_1+\lambda_2)$

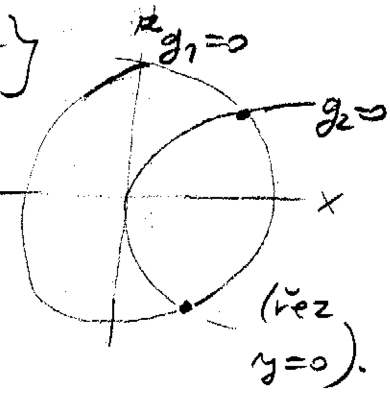
$4x(\lambda_1+\lambda_2) = 0$; máme $x = 0$;

2. rce: $\quad 0 = 2y(\lambda_1+\lambda_2) \to y = 0$

$(0,0,z) \notin M$ spor.

2.14. $f = x^2 + 2xz + y^2 + z$;

$M = \{\underbrace{x^2+y^2+z^2=1}_{\text{sféra}}\} \cap \{\underbrace{x = y^2+z^2}_{\text{paraboloid}}\}$



podezřelé body

(1) $h\begin{pmatrix} \nabla g_1 \\ \nabla g_2 \end{pmatrix} < 2$: $\begin{pmatrix} 2x, & 2y, & 2z \\ 1, & -2y, & -2z \end{pmatrix}$ $\quad \delta_1, \ \delta_2$

δ_1: $-4xy - 2y = 0$

$-2y(2x+1) = 0$

$x \geq 0$: máme $y = 0$; $\qquad$ δ_2: analogicky $z = 0$. spor.

(2) $\nabla f = \lambda_1 \nabla g_1 + \lambda_2 \nabla g_2$

$2x + 2z = \lambda_1 2x + \lambda_2$

$2y = \lambda_1 2y - \lambda_2 2y$

$2x + 1 = \lambda_1 2z - \lambda_2 2z$

2. rce: $\quad 2y(1 - \lambda_1 + \lambda_2) = 0$

(a) $y = 0$ nebo (b) $y \neq 0$; $\lambda_1 - \lambda_2 = 1$.

2.14 – pokr.:

(a) $y = 0$: rce 1 : $\left.\begin{aligned} x^2 + z^2 &= 1 \\ x &= z^2 \end{aligned}\right\}$ $x^2 + x - 1 = 0$; $x = \frac{-1 \pm \sqrt{5}}{2}$

$x \geq 0$; odtud $x = \frac{\sqrt{5}-1}{2}$; $z = \pm\sqrt{x}$

podezřelé $\left(\frac{\sqrt{5}-1}{2}, 0, \sqrt{\frac{\sqrt{5}-1}{2}}\right)$ $f \doteq 2.4$

$\left(\frac{\sqrt{5}-1}{2}, 0, -\sqrt{\frac{\sqrt{5}-1}{2}}\right)$. $f \doteq -1.37$

(b) $y \neq 0$; $\lambda_1 - \lambda_2 = 1$

3. rce $2x + 1 = 2z(\lambda_1 - \lambda_2) = 2z$

rovnice pro 1: $x^2 + x = 1$: $x = \frac{\sqrt{5}-1}{2}$;

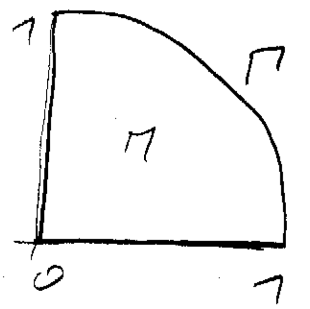
$z = x + \frac{1}{2} = \frac{\sqrt{5}}{2}$;

$y^2 = x - z^2 = \frac{\sqrt{5}-1}{2} - \frac{5}{4} = \frac{\sqrt{5}}{2} - \frac{7}{4} < 0$; spor

Závěr: výše uvedené podezřelé body jsou MAX / MIN.

2.15. $f = \operatorname{arctg} x + \operatorname{arctg} y$; $M = \{x^2+y^2 \le 1\} \cap \{x, y \ge 0\}$

podezřelé body



(1) uvnitř: $\nabla f = 0$ ..

$$\frac{\partial f}{\partial x} = \frac{1}{1+x^2} = 0 \text{ - nemá řešení}$$

(2) hranice : úsečky $(x,0)$; $x \in [0,1]$

f roste ; podezřelé $(0,0)$ $f = 0$

$(1,0)$ $f = \frac{\pi}{4} \doteq 0.79$

úsek $(0,y)$ .. podobně

$(0,1)$ $f = \frac{\pi}{4}$.

(3) úsek $\{x^2+y^2=1\} \cap \{x, y > 0\} =: \Gamma$

(i) $\nabla g = 0$ .. nikdy na Γ

(ii) $\nabla f = \lambda \nabla g$

$$\frac{1}{1+x^2} = \lambda \cdot 2x$$

$$\frac{1}{1+y^2} = \lambda\, 2y$$

$$\Rightarrow \quad 2y(1+y^2) = 2x(1+x^2)$$

$\rightarrow x = y$; neboť $t \mapsto t(1+t^2)$ roste na $(0,1)$)

$\Rightarrow \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ .. $f \doteq 1.23$

Závěr: $(0,0)$ MIN

$\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ MAX