Extrémy funkcí více proměnných

Úlohy řešitelné bez věty o Lagrangeových multiplikátorech

Nalezněte absolutní extrémy funkce f na množině M.

1.1. $f(x,y) = x + y;\ M = \{[x,y] \in \mathbb{R}^2;\ x^2 + y^2 = 1\}$

1.2. $f(x,y) = e^x;\ M = \{[x,y] \in \mathbb{R}^2;\ x^2 + 2y^2 = 1\}$

1.3. $f(x,y) = x^2 + y;\ M = \{[x,y] \in \mathbb{R}^2;\ x \geq 0, y \geq 0, x + y \leq 1\}$

1.4. $f(x,y) = x;\ M = \{[x,y] \in \mathbb{R}^2;\ 2 \geq x \geq 0, 1 \geq y \geq 0, 2x + y \leq 2\}$

1.5. $f(x,y,z) = (x+y)^2 + (x-y)^2 + z;\ M = [-1;1] \times [-1;1] \times [-1;1]$

1.6. $f(x,y) = \frac{x}{a} + \frac{y}{b},\ a > 0, b > 0;\ M = \{[x,y] \in \mathbb{R}^2;\ x^2 + y^2 \leq 1\}$

1.7. $f(x,y,z) = x^2 + y^2 + z^2 + 2x + 4y - 6z;\ M = \mathbb{R}^3$

1.8. $f(x,y) = (x^2 + y^2)e^{-(x^2+y^2)};\ M = \mathbb{R}^2$

1.9. $f(x,y,z) = x^2 + y^2 + z^2;\ M = \{[x,y,z] \in \mathbb{R}^3;\ \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \leq 1\};\ a > b > c > 0$

1.10. $f(x,y) = x^2 + y^2,\ M = \{[x,y] \in \mathbb{R}^2;\ x^2 + 4y^2 = 1\}$

1.11. Určete rozměry vodní nádrže ve tvaru kvádru o objemu $32m^3$ tak, aby dno a stěny měly dohromady nejmenší povrch.

1.12. $f(x,y,z) = 50x^{2/5}y^{1/5}z^{1/5};\ M = \{[x,y,z] \in \mathbb{R}^3;\ 80x + 12y + 10z = 24000,\ x > 0,\ y > 0,\ z > 0\}$

1.13. $f(x,y) = (x+y)e^{-2x-3y};\ M = \{[x,y] \in \mathbb{R}^2; x \geq 0, y \geq 0\}$

Úlohy na větu o Lagrangeových multiplikátorech

Řešte následující úlohy na absolutní extrémy funkcí.

2.1. $f(x,y,z) = xyz;\ M = \{[x,y,z] \in \mathbb{R}^3;\ x^2 + y^2 + z^2 = 1\}$

2.2. $f(x,y,z) = \sin x \sin y \sin z;\ M = \{[x,y,z] \in \mathbb{R}^3; x + y + z = \frac{\pi}{2}, x > 0, y > 0, z > 0\}$

2.3. $f(x,y,z) = xy^2z^3;\ M = \{[x,y,z] \in \mathbb{R}^3;\ x + 2y + 3z = a, x > 0,\ y > 0,\ z > 0\}$; kde $a > 0$

2.4. $f(x,y) = x + y;\ M = \{[x,y] \in \mathbb{R}^2;\ x^3 + y^3 - 2xy = 0, x \geq 0, y \geq 0\}$

2.5. $f(x,y) = y;\ M = \{[x,y] \in \mathbb{R}^2;\ (x^2 + y^2)^2 - 2(x^2 - y^2) = 0\}$

2.6. $f(x,y) = x^2 + y;\ M = \{[x,y] \in \mathbb{R}^2;\ 4y^3 - 4y + x^2 = 0,\ y \geq 0\}$

2.7 $f(x,y) = x^4y;\ M = \{[x,y] \in \mathbb{R}^2;\ x^4 + y^4 \leq 16, x \geq -1\}$

2.8. $f(x,y) = 2x + 4y;\ M = \{[x,y] \in \mathbb{R}^2;\ \sqrt[4]{x} + \sqrt[4]{y} \leq 1, x \geq 0,\ y \geq 0\}$

2.9. $f(x,y,z) = e^{-z^2}(x^2 + xy + y^2);\ M = \{[x,y,z] \in \mathbb{R}^3; x^2 + y^2 = 1, |z| \leq 1\}$

2.10. $f(x,y) = -y^2 + x^2 + \frac{4}{3}x^3;\ M = \{[x,y] \in \mathbb{R}^2;\ x^2 + y^2 \leq 4, x \leq 0\}$

2.11. $f(x,y,z) = xy + yz;\ M = \{[x,y,z] \in \mathbb{R}^3;\ x^2 + y^2 + z^2 = 1,\ x + y + z = 1\}$

2.12. $f(x,y) = (x^2 + 7y^2)e^{-(2x^2+y^2)};\ M = \{[x,y] \in \mathbb{R}^2;\ x^2 + 4y^2 \leq 1\}$

2.13. $f(x,y,z) = z + e^{xy};\ M = \{[x,y,z] \in \mathbb{R}^3;\ x^2 + y^2 + z^2 = 1,\ x^2 + y^2 = z^2\}$

2.14 $f(x,y,z) = x^2 + 2xz + y^2 + z;\ M = \{[x,y,z] \in \mathbb{R}^3;\ x^2 + y^2 + z^2 = 1,\ x = y^2 + z^2\}$

2.15 $f(x,y) = \operatorname{arctg} x + \operatorname{arctg} y;\ M = \{[x,y] \in \mathbb{R}^2;\ x^2 + y^2 \leq 1,\ x \geq 0, y \geq 0\}$