$$-4\int x^{-5}\ln(1+\sqrt{x^4+1})\,dx$$

Maximální otevřené intervaly $x \in (-\infty, 0)$ a $(0, \infty)$.

Per partes: $u' = -4x^{-5}$, tj. $u = x^{-4}$ a $v = \ln(1+\sqrt{x^4+1})$, tedy

$$v' = \frac{1}{1+\sqrt{x^4+1}}\frac{2x^3}{\sqrt{x^4+1}}$$

– dostaneme:

$$x^{-4}\ln(1+\sqrt{x^4+1}) - \underbrace{\int \frac{2x^3\,dx}{x^4\sqrt{x^4+1}(1+\sqrt{x^4+1})}}_{I}$$

Dále zpracujeme I substitucí $y = \sqrt{x^4+1}$, neboť

$$\frac{2x^3\,dx}{\sqrt{x^4+1}} = dy, \qquad x^2 = y^2 - 1$$

(nebo postupně, substitucí $x^4 = t$ a pak $y = \sqrt{t+1}$) a dostaneme

$$I = \int \frac{dy}{(y-1)(y+1)^2}$$

Rozklad na parciální zlomky

$$\frac{1}{(y-1)(y+1)^2} = \frac{A}{y-1} + \frac{B}{y+1} + \frac{C}{(y+1)^2}$$

a vyjde $A = 1/4$, $B = -1/4$, $C = -1/2$.

Po celkovém dosazení

$$-4\int x^{-5}\ln(1+\sqrt{x^4+1})\,dx$$
$$= x^{-4}\ln(1+\sqrt{x^4+1}) - \frac{1}{4}\ln(\sqrt{x^4+1}-1) + \frac{1}{4}\ln(\sqrt{x^4+1}+1) - \frac{1}{2}\frac{1}{\sqrt{x^4+1}+1}$$

v intervalech $(-\infty, 0)$ a $(0, \infty)$.

(2) $$\int \frac{dx}{\sin x\,(2+\cos x+\sin x)}$$
$x \in (k\pi, (k+1)\pi)$

$t = \operatorname{tg}\frac{x}{2}$

$\sin x = \frac{2t}{1+t^2}$ $\qquad dx = \frac{2\,dt}{1+t^2}$

$\cos x = \frac{1-t^2}{1+t^2}$

$$= \int \frac{1}{\frac{2t}{1+t^2}\cdot\left(2+\frac{1-t^2}{1+t^2}+\frac{2t}{1+t^2}\right)}\cdot\frac{2\,dt}{1+t^2} = \int g(t)\,dt;$$

$$g(t) = \frac{t^2+1}{t(t^2+2t+3)} = \frac{A}{t} + \frac{Bt+C}{t^2+2t+3};$$
$A = \frac{1}{3}$

$B = \frac{2}{3},\ C = -\frac{2}{3}$

$$\int g(t)\,dt = \underbrace{\int \frac{1}{3t}\,dt}_{\frac{1}{3}\ln|t|} + \frac{1}{3}\underbrace{\int\frac{2t-2}{t^2+2t+3}\,dt}_{I};$$

$$I = \underbrace{\int\frac{2t+2}{t^2+2t+3}\,dt}_{\ln(t^2+2t+3)} - 4\underbrace{\int\frac{dt}{t^2+2t+3}}_{J}$$

$$t^2+2t+3 = (t+1)^2+2 = 2\left[\left(\frac{t+1}{\sqrt2}\right)^2+1\right]$$

$$J = \frac{1}{\sqrt2}\operatorname{arctg}\left(\frac{t+1}{\sqrt2}\right);$$

$$\int g(t)\,dt = \frac{1}{3}\left\{\ln|t| + \ln(t^2+2t+3) + 2\sqrt2\operatorname{arctg}\left(\frac{t+1}{\sqrt2}\right)\right\}$$
$t \in (-\infty,0),\ (0,+\infty).$

uslov: $\uparrow$ $t = \operatorname{tg}\frac{x}{2}$; $x \in (k\pi,(k+1)\pi)$; $k \in \mathbb{Z}$