1a) $\operatorname{tg} x = x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + o(x^5)$, $x \to 0$

1b) $\operatorname{arctg} x = x - \frac{1}{3}x^3 + \frac{1}{5}x^5 + o(x^5)$, $x \to 0$

1c) $\arcsin x = x + \frac{1}{6}x^3 + \frac{3}{40}x^5 + o(x^5)$, $x \to 0$

1d) $\sinh x = x + \frac{1}{6}x^3 + \frac{1}{120}x^5 + o(x^5)$, $x \to 0$

1e) $\cosh x = 1 + \frac{1}{2}x^2 + \frac{1}{24}x^4 + o(x^5)$, $x \to 0$

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2a) $e^x - \sin x - 1 = \frac{1}{2}x^2 + o(x^2)$; $\quad \lim = \frac{1}{2}$

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2b) $$f(x) = \underbrace{\left(\frac{x}{\sin x}\right)^4}_{\to 1} \cdot \underbrace{\left(\frac{x^2}{\cos x - 1}\right)^2}_{\to 4} \cdot \frac{(e^{x^2}-1)(\sin x - x)^2}{x^8}$$

$(e^{x^2}-1) = x^2 + \frac{1}{2}x^4 + \frac{1}{6}x^6 + o(x^6)$

$(x - \sin x)^2 = \frac{1}{36}x^6 + o(x^6)$

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$(e^{x^2}-1)(x-\sin x)^2 = \frac{1}{36}x^8 + o(x^8)$; $\quad \lim = \frac{1}{9}$

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2c) $e^{\sin x} = 1 + x + \frac{1}{2}x^2 + o(x^3)$ $\quad \lim = \frac{1}{6}$

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2d) $\cos x - e^{-\frac{x^2}{2}} = -\frac{1}{12}x^4 + o(x^4)$; $\quad \lim = -\frac{1}{12}$

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2e) $\cos x - \cosh x = -x^2 + o(x^2)$;

$$f(x) = \underbrace{\frac{1}{x}}_{\to +\infty} \cdot \underbrace{(-1 + o(1))}_{\to -1} \to -\infty, \quad x \to 0+.$$

**2f)** $\sqrt{1+x^2} = \frac{1}{2}x^2 + o(x^2)$

$\sqrt[3]{1-x^2} = (1-x^2)^{\frac{1}{3}} = -\frac{1}{3}x^2 + o(x^2)$ $\qquad \lim = \frac{5}{6}$

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**2g)** $f(x) = \underbrace{\left(\frac{x}{\arcsin x}\right)^2}_{\to 1} \cdot \frac{\sqrt{\cos x} - 1}{x^2}$ $\qquad \lim = -\frac{1}{4}$

$\sqrt{\cos x} = 1 - \frac{1}{4}x^2 + o(x^2)$ ;

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**2h)** $\ln(\cos x) = o(x)$ , $x \to 0$ : $\qquad \lim = 0$

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**2i)** $\sqrt[3]{1+x} - \sqrt[5]{1-x} = \frac{8}{15}x + o(x)$ ;

$\sqrt[5]{1+x} - \sqrt[3]{1-x} =$ —"— ; $\qquad \lim = 1$

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**2j)** $\operatorname{tg} x - x = \frac{1}{3}x^3 + o(x^3)$,

$x - \sin x = \frac{1}{6}x^3 + o(x^3)$, $\qquad \lim = 2$

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**2k)** $\sqrt[3]{1+3x} - \sqrt[4]{1+4x} = \frac{1}{2}x^2 + o(x^2)$

$(a \neq b)$ $\cos ax - \cos bx = (b^2 - a^2)\frac{1}{2}x^2 + o(x^2)$ $\qquad \lim = \frac{1}{b^2 - a^2}$

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**2l)** $\sqrt{1+\operatorname{tg} x} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{11}{48}x^3 + o(x^3)$

$\sqrt{1+\sin x} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{48}x^3 + o(x^3)$ $\qquad \lim = \frac{1}{4}$

Pozn.: rozšířit $\sqrt{\quad} + \sqrt{\quad}$ by bylo snazší...