INTRODUCTION TO THE INTERPOLATION THEORY 1, NMMA533, WINTER TERM 2024–2025, EXAM REQUIREMENTS

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Required definitions

- operators: Laplace transform, Fourier transform, Riesz potential, dilation operator, Hardy averaging operator, Hardy–Littlewood maximal operator, Hilbert transform
- operator of weak type, operator of strong type
- distribution function, non-increasing rearrangement, maximal nonincreasing rearrangement, equimeasurable functions
- Lorentz functional and Lorentz space

Required theorems (without proofs)

- Laplace transform on L^2 (Theorem 1)
- Young's convolution theorem (Theorem 8)
- interpolation of compact operators (Theorem 9)
- basic estimates for f^* and f_* (Theorem 14)
- the Hardy–Littlewood inequality (Theorem 15)
- characterization of f^{**} (Theorem 16)
- elementary properties of Lorentz spaces (Theorem 17)
- Hanička's theorem (Theorem 18)
- Lorentz norms (Theorem 19)
- alternative norm in a Lorentz space (Theorem 23)

Required theorems (with proofs)

- embeddings of Lebesgue spaces (Theorem 2)
- Riesz's interpolation theorem for positive operators (Theorem 4)
- Riesz–Thorin interpolation theorem (Theorem 6)
- weak type estimate for the Riesz potential (Theorem 10)
- interpolation of weak-type operators in the diagonal case (Theorem 11)
- Vitali-type covering theorem (Theorem 12)
- weak-type estimate for the Hardy–Littlewood maximal operator (Theorem 13)
- embeddings of Lorentz spaces (Theorem 13)
- Marcinkiewicz's interpolation theorem (Theorem 24)

Date: 17. prosince 2024.