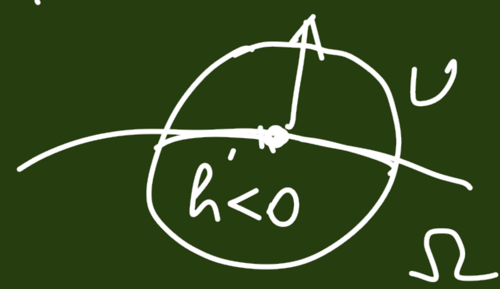
OPAKOVÁNÍ.

(M, ν) ... orientovaná $(n-1)$-plocha v $\mathbb{R}^n$, ν .. normála

- lokální popis $H(U)$ pomocí rozhraničující funkce



$h : U \to \mathbb{R}$, $h \in C^1(U)$, $\nabla h \neq 0$

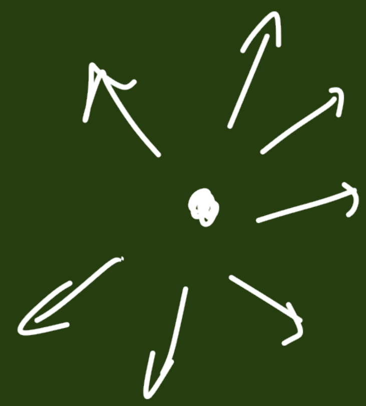
$$\nu = \frac{\nabla h}{\|\nabla h\|}$$

- $H_*(\Omega)$ .. reg. body hranice, $H_*(\Omega) \neq \emptyset \Rightarrow$ je to $(n-1)$-plocha
- koule v $\mathbb{R}^n$, $H(\Omega) = H_*(\Omega)$, čtverec v $\mathbb{R}^2$, $H(\Omega) \neq H_*(\Omega)$, $\mathcal{H}^1(H(\Omega) \setminus H_*(\Omega)) = 0$

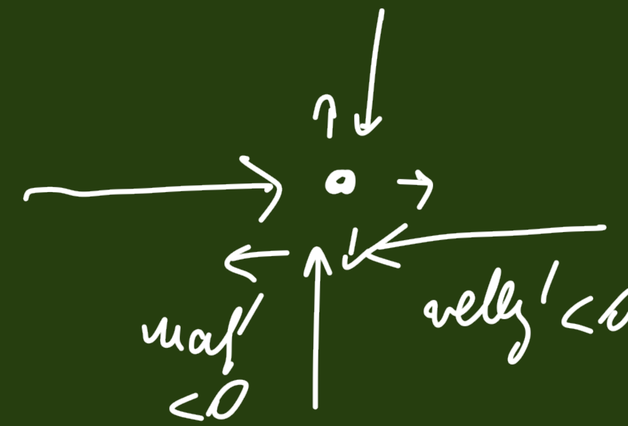
kychle, $\mathcal{H}^2(H(\Omega) \setminus H_*(\Omega)) = 0$.

- tok vektorového pole, divergence, Gaussova věta

zdroj
zřídlo
pramen
(source)

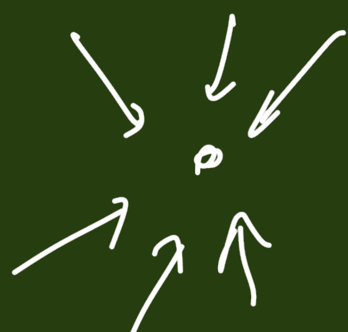


div $f > 0$



$f_1 \downarrow$

$\frac{\partial f_1}{\partial x_1} < 0$

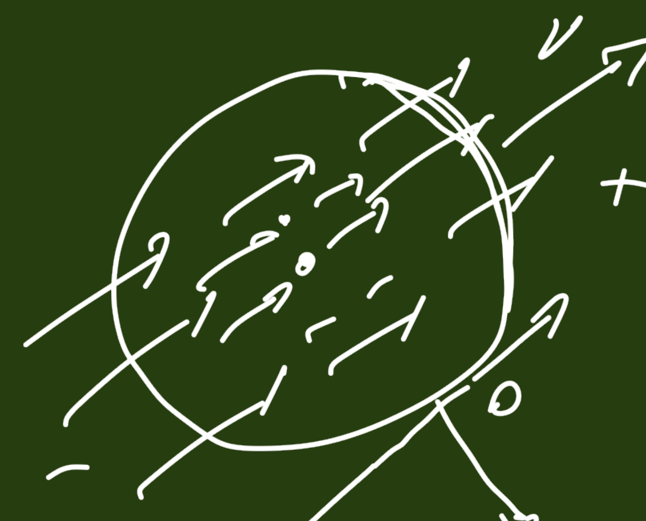


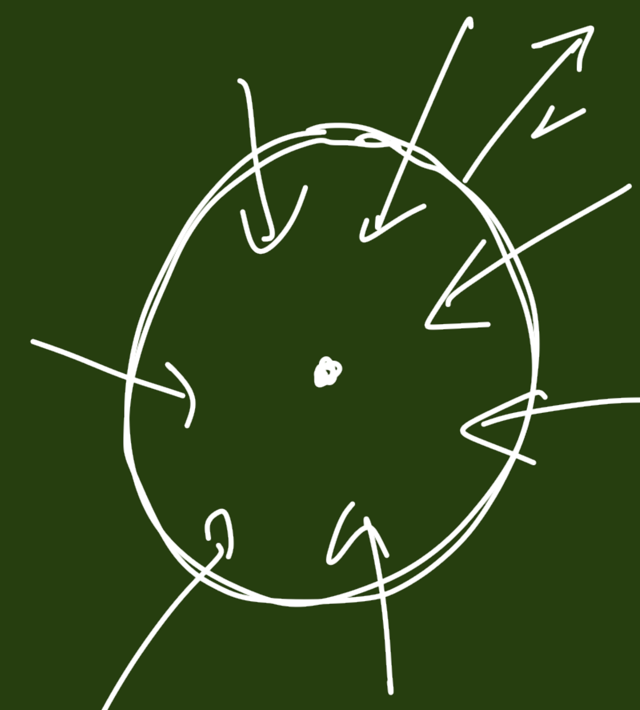
div $f < 0$

propad
výpusť
dřez

[sink]

div $f = 0$ ... bezzdrojové





orientace :
- $(n-1)$-plochy
- 1-plochy

DEF. $n>1$, M 1-plocha v $\mathbb{R}^n$,

orientace M ... spojité $\tau : M \to \mathbb{R}^n$, $\tau(x) \in T_x(M)$, $\|\tau(x)\| = 1$.

POZN. Pro $n=2$ máme $\nu(x)$ i $\tau(x)$.

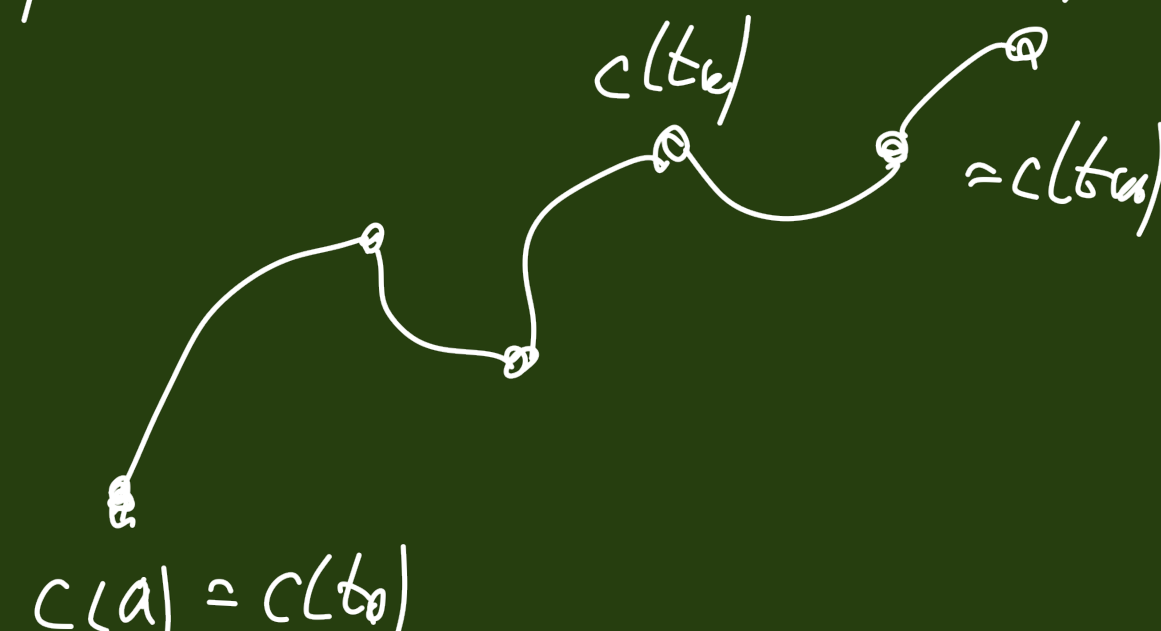
Mírně preferujeme τ.

POZN. křivka $\times$ 1-plocha

DEF. **křivka** $c : [a,b) \to \mathbb{R}^n$ spojitá

po částech reg. $c'(t) \neq 0$ vyjma konečně $t_0, \ldots, t_m$ (dělení $[a,b]$)

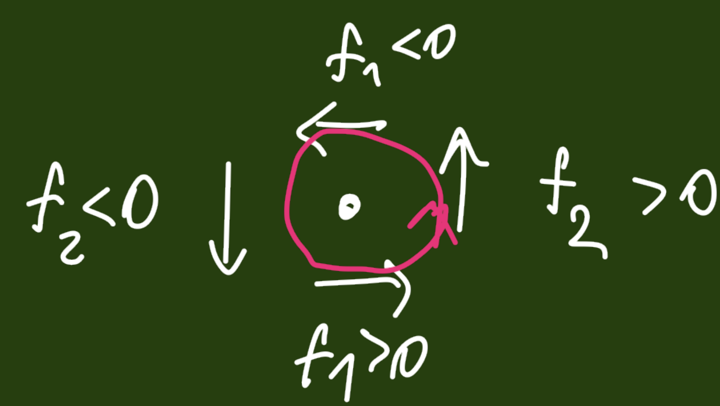
jedn., uz. : $c|_{[a,b)}$ prostá, $c(a) = c(b)$



DEFINICE. (a) Nechť $U \subset \mathbb{R}^2$ je otevřená, $f: U \to \mathbb{R}^2$, $f \in C^1(U)$ a $x \in U$. Rotace f je definována předpisem

$$\operatorname{curl} f(x) = \frac{\partial f_2}{\partial x_1}(x) - \frac{\partial f_1}{\partial x_2}(x) \quad \text{pro } x \in U.$$

(rotation, вихрь)



f_2 roste podél x_1

$$\frac{\partial f_2}{\partial x_1} > 0$$

f_1 klesá podél x_2

$$\frac{\partial f_1}{\partial x_2} < 0$$

$\Leftarrow$ $\operatorname{curl} f > 0$

(b) Nechť $U \subset \mathbb{R}^3$ je otevřená, $f: U \to \mathbb{R}^3$, $f \in C^1(U)$, $x \in U$. Rotace f je definována předpisem

$$\operatorname{curl} f(x) = \left(\frac{\partial f_3}{\partial x_2}(x) - \frac{\partial f_2}{\partial x_3}(x), \frac{\partial f_1}{\partial x_3}(x) - \frac{\partial f_3}{\partial x_1}(x), \frac{\partial f_2}{\partial x_1}(x) - \frac{\partial f_1}{\partial x_2}(x) \right).$$

Pozn. mnemotechnika: $\operatorname{curl} f = \nabla \times f = \begin{pmatrix} e^1 & \frac{\partial}{\partial x_1} & f_1 \\ e^2 & \frac{\partial}{\partial x_2} & f_2 \\ e^3 & \frac{\partial}{\partial x_3} & f_3 \end{pmatrix}$.

(pouze v $\mathbb{R}^3$!)

Pozn.
- $\operatorname{div}(\nabla) = \Delta$
- $\operatorname{curl} \operatorname{curl} = \nabla \operatorname{div} - \Delta$

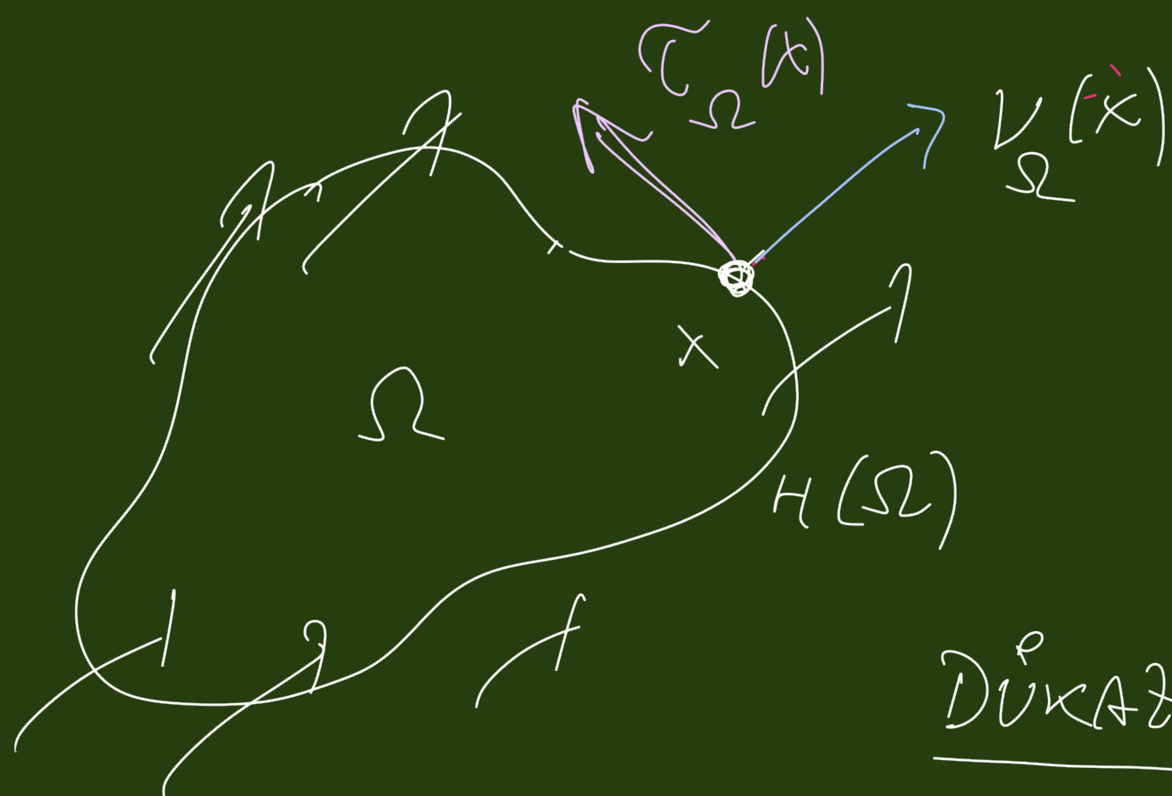
Věta 20.20 (Green) $\Omega \subset \mathbb{R}^2$ ot. om. $\neq \emptyset$, $\mathcal{H}^1(H(\Omega)) < \infty$, $\mathcal{H}^1(H(\Omega) \setminus H_*(\Omega)) = 0$,

$f : G \to \mathbb{R}^2$, G ot., $G \supset \overline{\Omega}$, $f \in \mathcal{C}^1(G)$,

$x \in H_*(\Omega)$: $\tau_\Omega(x) = -(\nu_\Omega(x)\ x)$

Potom

$$\int_{H(\Omega)} \langle f, \tau_\Omega \rangle \, d\mathcal{H}^1 = \int_\Omega \operatorname{curl} f \, d\lambda^2$$



Důkaz: $h = (f_2, -f_1)$, pak $\langle h, \nu_\Omega \rangle = \langle f, \tau_\Omega \rangle$ & $\operatorname{div} h = \operatorname{curl} f$. Tvrzení plyne z Gaussovy věty. $\square$