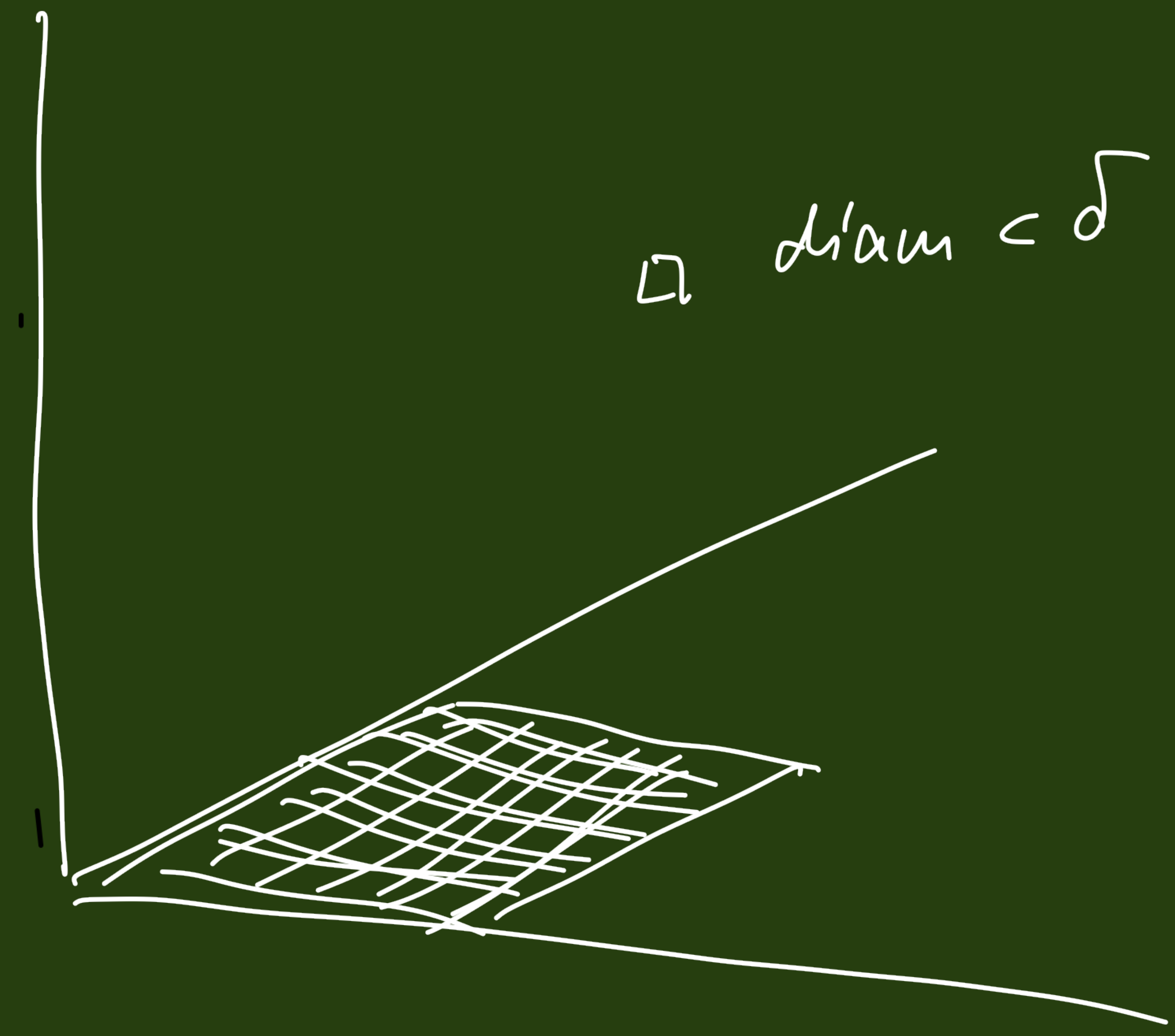
$\square$ diam $< \delta$



$$L : \mathbb{R}^k \to \mathbb{R}^m \quad \text{lin.}$$

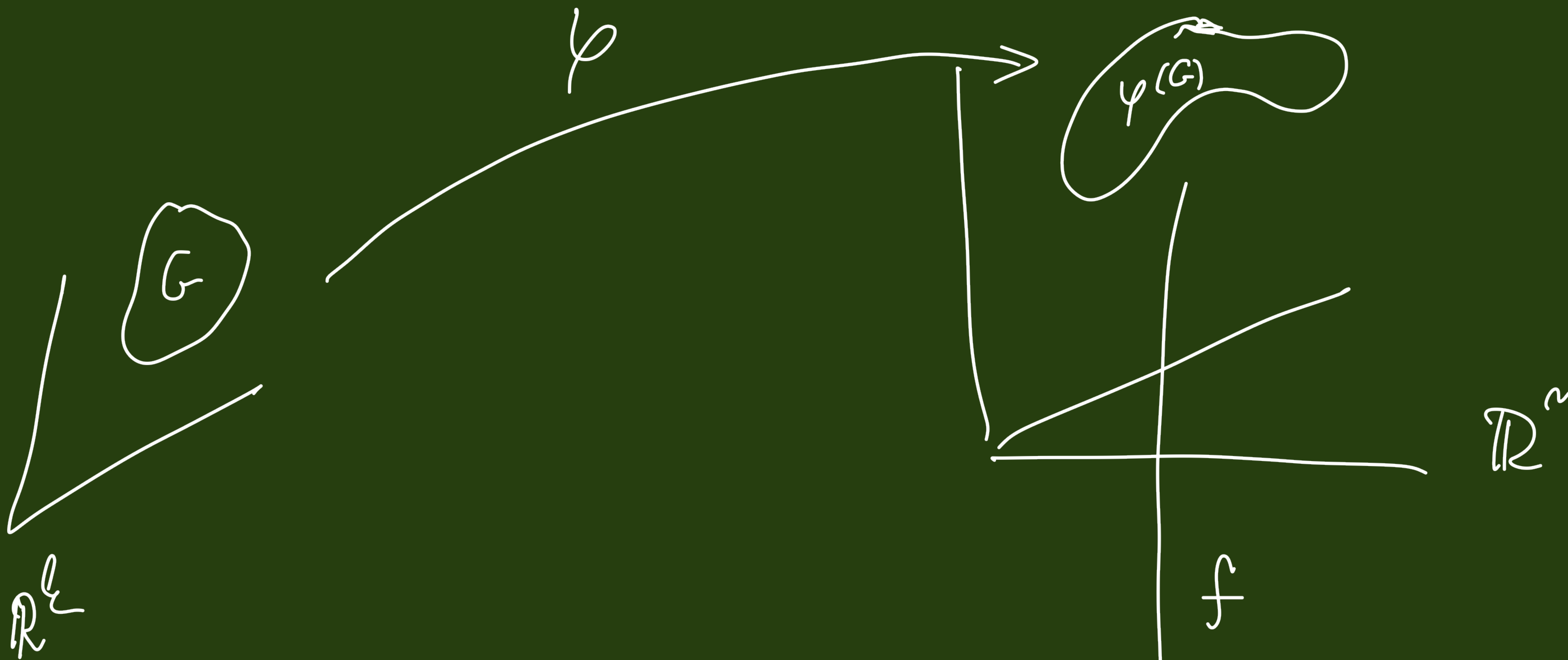
$$\ldots \text{ matice } (m \times k)$$

$$L^T \qquad \begin{array}{cc} & L^T \\ k \updownarrow & \begin{pmatrix} \cdots\cdots \\ \cdots\cdots \end{pmatrix} \\ & m \end{array} \begin{array}{cc} L & \\ \begin{pmatrix} \\ \\ \\ \\ \end{pmatrix} & \updownarrow m \\ \longleftrightarrow & \\ k & \end{array}$$

$$L^T \cdot L = \begin{array}{cc} \begin{pmatrix} \cdots \\ \cdots \\ \cdots \end{pmatrix} & \updownarrow k \\ \longleftrightarrow & \\ k & \end{array}$$

$$\det L^T L \geq 0$$

$$(L^T L x, x) = (Lx, Lx) \geq 0$$



$$\int_{\varphi(G)} f(x)\, d\mathcal{H}^k(x)$$

$$= \int_G f\big(\varphi(t)\big) \cdot \mathrm{vol}\, \varphi'(t)\, d\lambda^k(t)$$

$$\alpha_k = \frac{\Gamma\left(\frac{1}{2}\right)^k}{\Gamma\left(\frac{k}{2}+1\right) 2^k} = \frac{\pi^{k/2}}{\Gamma\left(\frac{k}{2}+1\right) 2^k}$$

PŘÍKLAD spočtěte „délku" oblouku C

$$C = \{ [x,y,z] \in \mathbb{R}^3, \ x = 3t, \ y = 3t^2, \ z = 2t^3 \}$$

mezi body

Tedy chceme $\mathcal{H}^1(C)$ $[0,0,0]$ a $[3,3,2]$,

Řešení. $t \in [0,1]$ ... není otevřené v $\mathbb{R}^1$

$C_1 = \{ [0,0,0], [3,3,2] \}$

$C = C_1 \cup C_2$

$C_2 = C \setminus C_1$

$n = 3, \ k = 1$

přímo z def. $\mathcal{H}^k$: $\quad \mathcal{H}^1(\{x\}) = 0 \Rightarrow \mathcal{H}^1(C_1) = 0$

<u>C_2</u>: $\quad G = (0,1)$ ot. v $\mathbb{R}^1$

$$\varphi : G \to \mathbb{R}^3 : \qquad \varphi(t) = \begin{pmatrix} 3t \\ 3t^2 \\ 2t^3 \end{pmatrix}$$

$$\varphi \text{ prosté, } C^1, \qquad \varphi'(t) = \begin{pmatrix} 3 \\ 6t \\ 6t^2 \end{pmatrix}$$

$f : \mathbb{R}^3 \to \mathbb{R}$, $\quad f \equiv 1$ (borelovská)

Tedy $$\mathcal{H}^1(C_2) = \int_{\varphi(G)} 1 \, d\mathcal{H}^1 \overset{AF}{=} \int_0^1 \underbrace{f(\varphi(t))}_{1} \cdot \mathrm{vol}\,\varphi'(t) \, dt$$

$$\operatorname{vol}\varphi'(t) = \sqrt{\det \varphi'(t)^T \varphi'(t)}$$

$$\varphi'(t)^T\varphi'(t) = (3, 6t, 6t^2)\begin{pmatrix}3\\6t\\6t^2\end{pmatrix} = 9+36t^2+36t^4$$

Także $\mathcal{H}^1(C_2) = \int_0^1 \sqrt{9+36t^2+36t^4}\,dt = 3\int_0^1(1+2t^2)\,dt$

$$= \underline{\underline{5}}$$