$\boxed{E4}$ $F: \mathbb{R}^2 \to \mathbb{R}^2$

$$F(x,y) = \left(\log(1+2x^2+3y^2)(|x|+|y|),\ \operatorname{sign}(x-y)\sqrt{3x^2+2y^2} \right)$$

(a) $F'(1,3)$,

(b) $F_1'(0,0)$,

(c) $\dfrac{\partial F_2}{\partial y}(0,0)$.

Řešení: (a) Pro $\delta \in (0,\sqrt{2})$ a $[x,y] \in B([1,3],\delta)$ platí

$$F(x,y) = \left(\log(1+2x^2+3y^2)(x+y),\ -\sqrt{3x^2+2y^2}\right).$$

Tedy $F \in C^1(B([1,3],\delta))$, a tedy $\forall [x,y] \in B([1,3],\delta)$ platí

$$F'(x,y) = \begin{pmatrix} \dfrac{4x(x+y)}{1+2x^2+3y^2} + \log(1+2x^2+3y^2) , & \dfrac{6y(x+y)}{1+2x^2+3y^2} + \log(1+2x^2+3y^2) \\ -\dfrac{3x}{\sqrt{3x^2+2y^2}} & \dfrac{-2y}{\sqrt{3x^2+2y^2}} \end{pmatrix},$$

takže

$$F'(1,3) = \begin{pmatrix} \dfrac{8}{15} + \log 30 , & \dfrac{12}{5} + \log 30 \\ -\dfrac{3}{\sqrt{21}} , & -\dfrac{6}{\sqrt{21}} \end{pmatrix}.$$

(b) Jest $F_1(x,y) = \log(1+2x^2+3y^2)(|x|+|y|)$. Tedy

$$\frac{\partial F_1}{\partial x}(0,0) = \lim_{x\to 0} \frac{F_1(x,0) - F_1(0,0)}{x}$$

$$= \lim_{x \to 0} \frac{|x| \cdot \log(1+2x^2)}{x} = 0,$$

$$\frac{\partial F_1}{\partial y}(0,0) = \lim_{y\to 0} \frac{F_1(0,y) - F_1(0,0)}{y}$$

$$= \lim_{y\to 0} \frac{|y| \log(1+3y^2)}{y} = 0 .$$

Kandidátem na $F_1'(0,0)$ je tedy $(0,0)$. Jest

$$\lim_{[x,y]\to[0,0]} \frac{|F_1(x,y) - F_1(0,0) - 0.x - 0.y|}{\sqrt{x^2+y^2}} =$$

$$= \lim_{[x,y]\to[0,0]} \frac{\left|\log(1+2x^2+3y^2)(|x|+|y|)\right|}{\sqrt{x^2+y^2}}$$

$$\leq \lim_{[x,y]\to[0,0]} \sqrt{2}. \left|\log(1+2x^2+3y^2)\right| = 0,$$

takže $F_1'(0,0)$ existuje a platí $F_1'(0,0) = 0$.

(c) Jest

$$\frac{\partial F_2}{\partial y}(0,0) = \lim_{y\to 0} \frac{F_2(0,y) - F_2(0,0)}{y} = \lim_{y\to 0} \frac{\operatorname{sign}(-y)\sqrt{2y^2}}{y}$$

$$= \lim_{y\to 0} -\sqrt{2}\frac{|y|.\operatorname{sign} y}{y} = -\sqrt{2} .$$

Hodnocení

(a) $F_2'(x,y)$ na okolí $[1,3]$ . . . 4

$F_2'(1,3)$ . . . 1

(b) $\nabla F_1(0,0)$ . . . 3

$F_1'(0,0)$ . . . 4

(c) $\frac{\partial F_2}{\partial y}(0,0)$ . . . 3