$\boxed{B4}$ $F: \mathbb{R}^2 \to \mathbb{R}^4$

$$F(x,y) = \left(\operatorname{arctg}(2x-y),\ x-y,\ (x^2-y^2)\frac{|x|}{1+|y|},\ e^{x+y} \right)$$

(a) $F(-1,1)$,

(b) $\frac{\partial F_3}{\partial x}(0,0)$.

<u>Řešení</u>. (a) Pro $x \in (-\infty, 0)$ a $y \in (0,\infty)$ platí

$$\frac{\partial F_1}{\partial x}(x,y) = \frac{2}{1+(2x-y)^2}, \quad \frac{\partial F_1}{\partial y}(x,y) = \frac{-1}{1+(2x-y)^2},$$

$$\frac{\partial F_2}{\partial x}(x,y) = 1, \quad \frac{\partial F_2}{\partial y}(x,y) = -1,$$

$$\frac{\partial F_3}{\partial x}(x,y) = \frac{-2x^2}{1+y} - \frac{x^2-y^2}{1+y},$$

$$\frac{\partial F_3}{\partial y}(x,y) = \frac{2xy}{1+y} + \frac{x(x^2-y^2)}{(1+y)^2},$$

$$\frac{\partial F_4}{\partial x}(x,y) = e^{x+y}, \quad \frac{\partial F_4}{\partial y}(x,y) = e^{x+y}.$$

Na množině $B([-1,1],1)$ jsou tedy všechny parciální derivace každé složky F spojité. Tedy existuje $F'(-1,1)$ a je reprezentována maticí

$$F'(-1,1) \approx \begin{pmatrix} \frac{1}{5} & -\frac{1}{10} \\ 1 & -1 \\ -1 & -1 \\ 1 & 1 \end{pmatrix}.$$

(b) $$\frac{\partial F_3}{\partial x}(0,0) = \lim_{t\to 0} \frac{F_3(t,0) - F_3(0,0)}{t}$$

$$= \lim_{t\to 0} \frac{t^2|t| - 0}{t} = 0.$$

Hodnocení:

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