$\boxed{B2}$ $$\int \frac{2e^{4x}-5e^{3x}+8e^{2x}-e^{x}}{(e^{2x}-2e^{x}-3)(e^{2x}-e^{x}+2)}\,dx =: I$$

Řešení: Substituce $y=e^x$, $dy=e^x dx$. Označ

$$J=\int \frac{2y^3-5y^2+8y-1}{(y^2-2y-3)(y^2-y+2)}\,dy .$$

Rozložíme integrand

$$\frac{2y^3-5y^2+8y-1}{(y^2-2y-3)(y^2-y+2)} = \frac{2y^3-5y^2+8y-1}{(y-3)(y+1)(y^2-y+2)}$$

$$= \frac{A}{y-3}+\frac{B}{y+1}+\frac{Cy+D}{y^2-y+2}.$$

Tedy

$$2y^3-5y^2+8y-1 = A(y+1)(y^2-y+2)+B(y-3)(y^2-y+2) + (Cy+D)(y-3)(y+1).$$

Pro $y=3$ dostaneme

$$54-45+24-1 = A\cdot 4\cdot 8,$$

$$32 = A\cdot 4\cdot 8, \text{ tedy } A=1.$$

Pro $y=-1$ dostaneme

$$-2-5-8-1 = B\cdot(-4)\cdot 4,$$

$$-16 = -16B, \text{ tedy } B=1.$$

Pro $y=0$ dostaneme

$$-1 = 2A - 6B - 3D = -4 - 3D,$$

$$3 = -3D, \qquad \text{tedy } D = -1.$$

Pro $y = 1$ dostaneme

$$2 - 5 + 8 - 1 = 4A - 4B + (C+D)(-2)\cdot 2,$$

$$4 = 4 - 4 - 4C + 4, \qquad \text{tedy } C = 0.$$

Tedy

$$\text{integrand } J = \frac{1}{y-3} + \frac{1}{y+1} - \frac{1}{y^2 - y + 2}.$$

Dále jest

$$\int \frac{dy}{y-3} \stackrel{c}{=} \log|y-3|, \quad \int \frac{dy}{y+1} \stackrel{c}{=} \log|y+1|,$$

$$\int \frac{dy}{y^2 - y + 2} = \int \frac{dy}{\left(y - \frac{1}{2}\right)^2 + \frac{7}{4}} =$$

$$= \frac{4}{7} \int \frac{dy}{\left[\frac{2}{\sqrt{7}}\left(y - \frac{1}{2}\right)\right]^2 + 1}$$

$$\stackrel{c}{=} \frac{4}{7} \operatorname{arctg}\left(\frac{2}{\sqrt{7}}\left(y - \frac{1}{2}\right)\right) \frac{\sqrt{7}}{2}$$

$$= \frac{2}{\sqrt{7}} \operatorname{arctg}\left(\frac{2}{\sqrt{7}}\left(y - \frac{1}{2}\right)\right),$$

$y \in (-\infty, -1)$, $y \in (-1, 3)$, $y \in (3, \infty)$.

Použijeme VOS2 pro

$(a,b) = (-\infty, \log 3)$ nebo $(\log 3, \infty)$,

$(\alpha, \beta) = (0, 3)$ nebo $(3, \infty)$,

$\varphi : (\alpha, \beta) \to \mathbb{R}$, $\varphi(t) = \log t$,

potom
- $\varphi((\alpha, \beta)) = (a, b)$,
- $\varphi^{-1}(x) = e^x$, $x \in (a,b)$,
- $\varphi'(t) = \frac{1}{t}$ existuje vlastní a nenulová $\forall t \in (\alpha, \beta)$.

Tedy

$$I \overset{c}{=} \log|e^x - 3| + \log(e^x + 1) - \frac{2}{\sqrt{7}} \operatorname{arctg}\left(\frac{2}{\sqrt{7}}\left(e^x - \frac{1}{2}\right)\right),$$

$x \in (-\infty, \log 3)$ nebo $x \in (\log 3, \infty)$.

Hodnocení:

substituce	2
rozklad	3
integrace parc. zlomků	3
závěr	2