$\boxed{A4}$ $F : \mathbb{R}^2 \to \mathbb{R}^2$

$$F(x,y) = \begin{cases} \left(e^{x^2-y^2},\ y^2 \operatorname{arctg}\left(\frac{1}{x^2+y^2}\right)\right), & [x,y] \neq o \\ (0,0), & [x,y] = o. \end{cases}$$

(a) $F'(1,1)$, matici $J_F(-1,1)$

(b) $\frac{\partial F_2}{\partial y}(0,0)$.

Řešení. (a) Pro $[x,y] \neq o$ platí

$$\frac{\partial F_1}{\partial x} = 2x e^{x^2-y^2}, \qquad \frac{\partial F_1}{\partial y} = -2y e^{x^2-y^2},$$

$$\frac{\partial F_2}{\partial x} = y^2 \frac{1}{1+\frac{1}{(x^2+y^2)^2}} \cdot \frac{-2x}{(x^2+y^2)^2} =$$

$$= \frac{-2xy^2}{1+(x^2+y^2)^2},$$

$$\frac{\partial F_2}{\partial y} = 2y \operatorname{arctg} \frac{1}{x^2+y^2} + \frac{y^2}{1+\frac{1}{x^2+y^2}} \cdot \frac{-2y}{(x^2+y^2)^2}$$

$$= 2y \operatorname{arctg} \frac{1}{x^2+y^2} - \frac{2y^3}{1+(x^2+y^2)^2}.$$

Protože jsou všechny parciální derivace každé složky F spojité funkce na okolí $[1,1]$, existuje $F'(1,1)$ a je reprezentována maticí

$$\begin{pmatrix} \frac{\partial F_1}{\partial x}(1,1) & \frac{\partial F_1}{\partial y}(1,1) \\ \frac{\partial F_2}{\partial x}(1,1) & \frac{\partial F_2}{\partial y}(1,1) \end{pmatrix} = \begin{pmatrix} 2 & -2 \\ -\frac{2}{5} & 2\operatorname{arctg}\frac{1}{2} - \frac{2}{5} \end{pmatrix}.$$

Tedy

$$J_F = 4\operatorname{arctg}\frac{1}{2} - \frac{4}{5} - \frac{4}{5} = 4\operatorname{arctg}\frac{1}{2} - \frac{8}{5}.$$

(b) $$\frac{\partial F_2}{\partial y}(0,0) = \lim_{t\to 0} \frac{F_2(0,t) - F_2(0,0)}{t} =$$

$$= \lim_{t\to 0} \frac{t^2 \operatorname{arctg}\frac{1}{t^2} - 0}{t}$$

$$= \lim_{t\to 0} t \cdot \operatorname{arctg}\frac{1}{t^2} = 0.$$

Hodnocení (a) parciální derivace F v $[1,1]$ .. 2

existence $F'(1,1)$ .. 2

reprezentace $F'(1,1)$ .. 2

jacobián $J_F(1,1)$ .. 1

(b) $\frac{\partial F_2}{\partial y}(0,0)$ .. 3