POSLOUP.

Věty: ① Omezená mono má limitu. Opakování před. (kreslete si obrázky.) 1'

Detailněji: Shora om. neklesající má limitu.
Zdola om. nerostoucí má limitu

② Z omezené lze vybrat konvergentní! (Bolzano–Weierstrass, půlení)
"mající limitu."

!③ $f \le g$ na $P_r(a)$ $\wedge \exists \lim_{x\to a} f, \lim_{x\to a} g \Rightarrow \lim_{x\to a} f \le \lim_{x\to a} g$

④ Bolzano–Cauchy: $\forall \varepsilon > 0\ \exists n_0\ \forall n, m \ge n_0\ |a_n - a_m| < \varepsilon$.

Pak $\exists \lim_{n\to\infty} a_n$. (I $\Leftarrow$)

(Někdy se tomu říká věta o úplnosti $\mathbb{R}$. „Co se blíží má limitu.")

⑤ $\lim_{x\to a^+} f(x) = \inf_{x\in(a,b)} f(x)$ $\quad \lim_{x\to b^-} f(x) = \sup_{x\to(a,b)} f(x)$, je-li f neklesající.

↑ „intuitivní"!!!

$\lim_{x\to a^+} f(x) = \sup_{x\in(a,b)} f(x)$ $\quad \lim_{x\to b^-} f(x) = \inf_{x\to(a,b)} f(x)$, je-li f nerostoucí.

Př.: $\lim_{x\to 0^+} \frac{1}{x} = \infty$, $\lim_{x\to\infty} = \inf\left(\frac{1}{x}\right) = 0$

f nerost, $f(x) = \frac{1}{x}$, $x \in \mathbb{R}^+$

⑥ Heine: $a \in \mathbb{R}^*$, $A \in \mathbb{R}^*$.

$$\lim_{x\to a} f(x) = A \iff \forall (a_n)_n \quad a_n \to a,\ n\to\infty,\ a_n \neq a \text{ (pro } n \ge n_0)$$

$$\lim_{n\to\infty} f(a_n) = A.$$

Př. Limita posloupnosti

1. $$\lim_{n\to\infty}\frac{\sqrt{n^3-2n^2+1}+\sqrt[3]{n^4+1}}{\sqrt{n^6-6n^5+2}+\sqrt[5]{n^7+n^3+1}} \overset{\text{„triv." Heine nebodef}}{=} \lim_{x\to\infty}\frac{\sqrt{x^3-2x^2+1}+\sqrt[3]{x^4+1}}{\sqrt{x^6-6x^5+1}+\sqrt[5]{x^7+x^3+1}}$$

mocniny: $\boxed{\frac{3}{2}}$, $\boxed{\frac{4}{3}}$, $\boxed{3}$, $\boxed{7/5}$ — 0?

$$= \lim_{x\to 0^+}\frac{\sqrt{\frac{1}{x^3}-\frac{2}{x^2}+1}+\sqrt[3]{\frac{1}{x^4}+1}}{\sqrt{\frac{1}{x^6}-\frac{6}{x^5}+1}+\sqrt[5]{\frac{1}{x^7}+\frac{1}{x^3}+1}}\cdot\frac{x^3}{x^3} =$$

$$\frac{n^{\frac{3}{2}}}{n^3}=\frac{1}{n^{3/2}}$$

$$= \lim_{x\to 0^+}\frac{\sqrt{x^3-2x^4+x^6}+\sqrt{x^5+x^9}}{\sqrt{1-6x+x^6}+\sqrt{x^8+x^{12}+x^{15}}} = \frac{0+0}{1+0} = \underline{0}$$

2. $\lim_{n\to\infty}\frac{a^n}{n!}$, $a\in\mathbb{R}^+$ $\quad \lim_{n\to\infty}\frac{a^n}{n!}=\lim_{n\to\infty}\left(\frac{a}{n}\,\frac{a}{n-1}\cdots\frac{a}{1}\right)$; Zvol $\varepsilon<1$.

$b_n := \frac{a^n}{n!}$ $\quad$ Najdi $n_0\in\mathbb{N}$, že $\frac{a}{n_0}<\varepsilon$ $\quad \forall n\geq n_0 \ \frac{a}{n}<\varepsilon$

$$b_n \leq \frac{a^n}{n!} = \frac{a}{1}\cdots\frac{a}{n_0-1}\,\varepsilon^{n-n_0+1} \qquad \lim_{n\to\infty} b_n \leq \frac{a}{1}\cdots\frac{a}{n_0-1}\lim_{n\to\infty}\varepsilon^{n-n_0+1} = \frac{a}{1}\cdots\frac{a}{n_0-1}\,0 = 0.$$

3. $$\lim_{n\to\infty}\sqrt[n]{n} = \lim_{n\to\infty} n^{\frac{1}{n}} \overset{\text{Heine}}{=} \lim_{x\to\infty} x^{\frac{1}{x}} = e^{\lim_{x\to\infty}(\ln x)\frac{1}{x}} = e^0 = 1$$

$\left(\frac{\ln x}{x}\to 0, x\to\infty\right.$ růstová ř.

nebo l'Hosp.$\left.\right)$

4. $$\lim_{n\to\infty}\left[\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\cdots+\frac{1}{n(n+1)}\right] = \lim_{n\to\infty}\left[\left(\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\cdots+\left(\frac{1}{n}-\frac{1}{n+1}\right)\right] = \lim\left(\frac{1}{1}-\frac{1}{n+1}\right) = 1$$

1. $\lim\limits_{n\to\infty} \sqrt[n]{3} = \lim\limits_{n\to\infty} 3^{\frac{1}{n}} = \lim\limits_{n\to\infty} e^{\frac{1}{n}\ln 3} = e^{\lim\limits_{n\to\infty}\frac{\ln 3}{n}} = e^0 = 1$

2. $\lim\limits_{n\to\infty} \sqrt[n]{3^n+5^n} \Rightarrow e^{\boxed{\frac{\ln(3^n+5^n)}{n}}}$ $\quad \frac{\ln 5^n(1+(\frac{3}{5})^n)}{n} = \frac{n\ln 5 + \ln(1+\overbrace{(\frac{3}{5})^n}^{\to 0})}{n} \to \ln 5$

$= \ln 5 + \frac{\ln(1+(\frac{3}{5})^n)}{n} \xrightarrow[n\to\infty]{} \ln 5 + \frac{0}{\infty} = \ln 5 \Rightarrow e^{\ln 5} = 5$

Nebo / obdobně: Strážníci $\lim\limits_{n\to\infty} \sqrt[n]{3^n+5^n} = 5$: $5 = \sqrt[n]{5^n} \le \sqrt[n]{3^n+5^n} \le \sqrt[n]{5^n+5^n} = \sqrt[n]{5}\,5$

$\downarrow$ 5 $\qquad \downarrow$ $1\cdot 5$

$\frac{a^x}{x^B}$ $a>1$

3. $\lim\limits_{n\to\infty} \frac{3^n+5^n}{n!+6^n} = \lim\limits_{n\to\infty} \frac{\frac{3^n}{n!}+\frac{5^n}{n!}}{1+\frac{6^n}{n!}} = \frac{0+0}{1+0} = 0$

4. $\lim\limits_{n\to\infty} \frac{3^n+5^n}{7^n+6^n} = \lim\limits_{n\to\infty} \frac{(\frac{3}{7})^n+(\frac{5}{7})^n}{1+(\frac{6}{7})^n} = \frac{0+0}{1+0} = 0$

Výhodné pro faktoriály $a_n > 0$ $\quad \lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = L \begin{cases} <1 & \lim a_n = 0 \\ =1 & \text{nevíme} \\ >1 & \lim a_n = +\infty \end{cases}$

Dk.: 1. $0 \le \frac{a_{n+1}}{a_n} < L+\varepsilon \overset{<1}{\Rightarrow} a_{n+1} < (L+\varepsilon)a_n <$

$\dots < \underline{(L+\varepsilon)^n}_{<1}\, a_0 \to 0$

3. $1 < L-\varepsilon < \frac{a_{n+1}}{a_n}$ obdobně.

5. $\lim\limits_{n\to\infty} \frac{3^n}{n!}$ $\quad \frac{\frac{3^{n+1}}{(n+1)!}}{\frac{3^n}{n!}} = \frac{3}{n+1} \to 0 \overset{<1}{}$ $\quad \lim\limits_{n\to\infty} \frac{3^n}{n!} = 0$

6. $\lim \frac{(2n)!}{(n!)^2}$ $\quad \frac{(2n+2)!\; n!\,n!}{(n+1)!\,(n+1)!\,(2n)!} = \frac{(2n+2)(2n+1)}{(n+1)^2} \to 4 \overset{>1}{\Rightarrow}$

$\frac{4n^2+8n+2}{n^2+2n+1} = \frac{4+\frac{8}{n}+\frac{2}{n^2}}{1+\frac{2}{n}+\frac{1}{n^2}}$

$\Rightarrow \lim \frac{(2n)!}{(n!)^2} = +\infty$

$= \lim_{n\to\infty} \left[\frac{1}{1} - \frac{1}{n+1} \right] = 1.$

Př.: Pro která $\cancel{x} \in \mathbb{R}$ $\exists \lim_{n\to+\infty} (\sin nx)$.

1. $\cancel{x} = 1$ (~~k~~ zapamatování): $\lim_{n\to\infty} \sin \cancel{n} \nexists$.

Dk (Heine): $\alpha_{\cancel{n}} = n\pi$ $\quad \alpha_n \to \infty, n \to \infty$

$\beta_n = 2n\pi + \frac{\pi}{2}$ $\quad \beta_n \to \infty, n \to \infty$

$\sin \alpha_n = 0$

$\sin \beta_n = 1$

$\lim_{n\to\infty} \sin n\pi = 0$

$\lim_{n\to\infty} \sin\left(\frac{\pi}{2} + 2\pi n\right) = 1$ $\quad \lim_{n\to\infty} \sin n \nexists$

2. $x \in \mathbb{R}$. Nechť $\exists \lim_{n\to\infty} \sin(nx) = L$. Trik

dok

$\{\sin[(n+m)x] - \sin(nx)\cos(mx)\}^2 = \sin^2 mx \cos^2 nx$

$\downarrow n\to\infty$

$[L - L\cos(mx)]^2 = \sin^2(mx)(1-L^2) \quad \forall m$

$L^2(1-\cos mx)(1-\cos mx) = (1-\cos mx)(1+\cos mx)(1-L^2)$

a) $\cos mx \neq 1$ $\quad L^2(1-\cos mx) = (1+\cos mx)(1-L^2)$

$L^2 = 1 + \cos mx - L^2$

$\cos mx = 2L^2 - 1 \quad \forall m \; \forall x$

$-m \sin mx = 0 \iff mx = 2k\pi$, tj.

$\cos(2k\pi) = (-1)^k = 2L^2 - 1$

a)

Nechť $\lim_{n\to\infty} \sin(nx) = L$ existuje. Trik:

$$\to \{\sin[(n+m)x] - \sin(nx)\cos(mx)\}^2 = \sin^2(mx)\cos^2(nx)$$

$$\downarrow \qquad\qquad \downarrow$$

$$[L - L\cos(mx)]^2 = \sin^2(mx)(1-L^2)\ \forall m$$

$$L^2(1-\cos mx)(1-\cos mx) = (1-\cos mx)(1+\cos mx)(1-L^2)$$

1. Předpokl.: $\cos(mx) \neq 1$ $\quad L^2(1-\cos mx) = (1+\cos mx)(1-L^2)$

$$L^2 - \cancel{L^2\cos mx} = 1+\cos(mx) - L^2 - \cancel{L^2\cos(mx)}$$

$$\cos(mx) = 2L^2 - 1 \quad \forall m$$

- $\Rightarrow m\sin(mx) = 0 \Rightarrow x = \pi k \Leftrightarrow \underline{L = 0}$
- $\cos mx = 1 \Leftrightarrow mx = 2k\pi \Rightarrow$

Limes superior a inferior — nečíst: viz násl. strana

$$\limsup_{n\to\infty} a_n = \lim_{n\to\infty} (\sup_{n\geq k} a_n), \quad \liminf_{n\to\infty} a_n = \lim_{n\to\infty} (\inf_{n\geq k} a_n)$$

1. Najděte $\limsup a_n$, $\liminf a_n$

$$a_n = \frac{n-1}{n+1}\cos\left(\frac{2}{3}n\pi\right), \quad \frac{n-1}{n+1} = 1 - \frac{2}{n+1} \longrightarrow 1 \ !$$

$$\cos\left(\frac{2}{3}\pi n\right) \in \left\{\cos\frac{2}{3}\pi, \cos\frac{4}{3}\pi, \cos 2\pi\right\} = \left\{-\frac{1}{2}, 1\right\} \quad \forall n$$

$$\{$$

2. $a_n = \cos^n\left(\frac{2}{3}\pi n\right) \left\langle \begin{array}{ll} 1 & n = 3k \longrightarrow 1 \quad \limsup a_n = 1 \\ \left(-\frac{1}{2}\right)^n & \text{jindy} \longrightarrow 0 \quad \liminf a_n = 0 \end{array}\right.$

$\nabla\ \nabla$ Používám $\limsup_{n\to\infty} a_n = \sup (\lim_{k\to\infty} a_{n_k})$ a $\liminf a_n =$

$$= \inf(\lim_{k\to\infty} a_{n_k}) = 0$$

Limes superior a inferior

$$\limsup_{n\to\infty} a_n := \lim_{k\to\infty}\Big(\underbrace{\sup_{n\ge k} a_n}_{b_k}\Big) \quad \text{lim nerostoucí}$$

$$\liminf_{n\to\infty} a_n := \lim_{k\to\infty}\Big(\inf_{n\ge k} a_n\Big) \quad \text{lim neklesající}$$

Věta: $\limsup_{n\to\infty} a_n = \sup_{(n_k)_k}\big(\lim_{k\to\infty} a_{n_k}\big)$, pro $\lim_{k\to\infty} a_{n_k}$ $\exists$

$\liminf a_n = \inf_{(n_k)_k}\big(\lim a_{n_k}\big)$ pro $\lim_{k\to\infty} a_{n_k}$ $\exists$

(Množinově: $\sup\{\lim_{k\to\infty} a_{(n_k)} \mid (n_k)$ rostoucí & $\lim_{k\to\infty}(a_{n_k})$ existuje$\}$)

n_k rostoucí říkám a_{n_k} vybraná z a_n

Př.: Najdi $\limsup a_n$, $\liminf a_n$, $a_n = \frac{n-1}{n+1}\cos\left(\frac{2}{3}\pi n\right)$.

$n=3k$, $\cos(2\pi k)=\cos(0)=1$

$n=3k+1$, $\cos(2\pi k+\frac{2}{3}\pi)=\cos\frac{2}{3}\pi=-\frac{1}{2}$

$n=3k+2$, $\cos(\frac{4}{3}\pi)=-\frac{1}{2}$

$A \subseteq B$

$\inf A \ge \inf B$

$\sup A \le \sup B$

$1 \leftarrow \frac{3k-1}{3k+1}\cdot 1$, $\frac{3k}{3k+2}\cdot\left(\frac{-1}{2}\right) \to -\frac{1}{2}$

$\limsup = 1$

$\liminf = -\frac{1}{2}$

Př.: Dtto $a_n = \cos^n\left(\frac{2}{3}\pi n\right)$.

$\limsup = 1$

$\liminf = 0$

$1^{3k} = 1 \to 1$

$\left(-\frac{1}{2}\right)^{3k+1} = -\frac{1}{2}\left(-\frac{1}{2}\right)^{3k} \to 0$

$\left(-\frac{1}{2}\right)^{3k+2} = \frac{1}{4}\left(-\frac{1}{2}\right)^{3k} \to 0$

Př.: $a_n = \begin{cases} \frac{1}{n} & n=2k \\ 0 & n=2k+1 \end{cases}$ obě 0

Př. $a_n = \begin{cases} \frac{1}{2n} & 2k \\ \frac{1}{n} & 2k+1 \end{cases}$ obě 0

Př. $a_n = \begin{cases} \frac{1}{n} & n \text{ sudé} \\ \frac{(-1)^n}{n} & n \text{ liché} \end{cases}$ obě 0

Taylorův polynom

· Nechť $f \in C^k(I)$, $a \in I$, pak $T_k^a(f) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots + \frac{f^{(k)}(a)}{k!}(x-a)^k$ slují Taylorův polynom f se středem a stupně k, $k \in \mathbb{N}_0$.

Př.: Tayl. $f(x) = \sin x$, $\overset{a}{x} = 0$, $k=3$.

$\sin(0)=0$, $\sin'(0)=\cos(0)=1$, $\sin''(0)=\cos'(0)=-\sin(0)=0$, $\sin'''(0)=-\sin'(0)=-\cos(0)=-1$

$$T_3^0 \sin(x) = 1\cdot x - \frac{1}{3!}x^3 = \boxed{x - \frac{x^3}{6}}.$$

Př.: Urči $T_4^{\overset{=a}{0}}(\cos)$.

sami

$\cos(0)=1$, $\cos'(0)=-\sin(0)=0$, $\cos''(0)=-\cos(0)=-1$, $\cos'''(0)=0$, $\cos^{(IV)}(0)=1$

$$(T_4^0 \cos)(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!}$$

Věta o Tayl. pol.

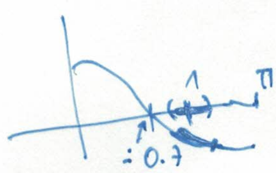
$$\cos x = 1 - \frac{x^2}{2} + o(x^2) \to o(x^3),\ x \to 0$$

$$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{4!} + o(x^4) \to o(x^5),\ x \to 0$$

Př.: $T_2^1 \cos$?

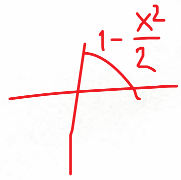
sami

$$(T_2^{\overset{a}{1}} \cos)(x) = \cos(1) + \frac{\overset{=\cos'(1)}{-\sin(1)}}{1!}(x-1)^1 - \frac{\overset{=\cos''(1)}{\cos(1)}}{2!}(x-1)^2.$$



Další Taylory: mocnina, logaritmus.

$$T_k^0 e^x = \sum_{n=0}^{k} \frac{x^n}{n!}$$



Sčítání: $f(x) = T_f^a + o(x^n)$

$g(x) = T_g^a + o(x^m)$

$f(x) + g(x) = T_f^a + T_g^a + o(x^{\min\{m,n\}})$

Násobení: $o(x^n)\, o(x^m) = o(x^{n+m})$, avšak

$x\, o(x^n) = o(x^{n+1})$: $\lim_{x\to 0} \frac{x f}{x^{n+1}} = \lim_{x\to 0} \frac{f}{x^n} = 0$

$\lim_{x\to 0} \frac{fg}{x^n x^m} = \lim \frac{f}{x^n} \lim \frac{g}{x^m} = 0 \cdot 0 = 0$

Př.: Tayl. pol. spočtěte limity

$1 + y + \frac{y^2}{2} +$

2. $\lim_{x\to 0} \frac{\cos x - e^{-\frac{x^2}{2}}}{x^4} = \lim_{x\to 0} \frac{1 - \frac{x^2}{2} + \frac{x^4}{4!} + o(x^4) - \left(1 - \frac{x^2}{2} + \frac{x^4}{4}\frac{1}{2} + o(x^4)\right)}{x^4}$

$4! = 6 \cdot 4 = 24$

$\frac{1}{24} - \frac{1}{8} = -\frac{1}{12}$

$= \lim_{x\to 0} \frac{-\frac{1}{12}x^4 + o(x^4)}{x^4} = -\frac{1}{12} + \lim_{x\to 0} \frac{o(x^4)}{x^4} = -\frac{1}{12}$

1. $\lim_{x\to 0} \frac{e^x \sin x - x(x+1)}{x^3} = \lim_{x\to 0} \frac{1}{x^3}\left[\left(1 + x + \frac{x^2}{2} + \frac{x^3}{6} + o(x^3)\right)\left(x - \frac{x^3}{3!} + o(x^3)\right)\right.$

$\left.- x(x+1)\right] = \lim_{x\to 0} \frac{1}{x^3}\left[x - \frac{x^3}{6 = 3!} + x^2 - \boxed{\frac{x^4}{6}} + o(x^3) + \frac{x^3}{2} + o(x^3) - x^2 - x\right] =$

$= \lim_{x\to 0} \frac{x^3}{x^3}\left(\frac{1}{2} - \frac{1}{3!}\right) + \lim \frac{o(x^3)}{x^3} = \frac{1}{3}.$

3. $\lim_{x\to 0} \frac{a^x + a^{-x} - 2}{x^2}$, $a > 0$ $\quad (a^x)' = \ln a\, a^x \quad (a^x)^{(k)} = (\ln a)^k a^x$

$\lim_{x\to 0} \frac{1}{x^2}\left[1 + \ln a\, a^x x + \ln^2 a\, a^x \frac{x^2}{2} + o(x^2) + 1 - \ln a\, a^x x + \ln^2 a\, a^x \frac{x^2}{2}\right.$

$+ o(x^2) - 2 \Big] = \lim\limits_{x\to 0} (\ln a)^2 a^x + \lim\limits_{x\to 0} \frac{o(x^2)}{x^2} =$

$= (\ln a)^2 a^x$. Zde obdobně l'Hospitalem.

Př: Spočtěte přibližně $\sqrt[5]{250}$.

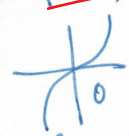
Znám: $(1+x)^\alpha = \sum\limits_{k=1}^{n} \binom{\alpha}{k} x^k + o(x^n)$ (Δ), $x \in (-1,1)$

$\sqrt[5]{250} = \sqrt[5]{\frac{250}{3^5}\cdot 3^5} = 3\sqrt[5]{\frac{250}{243}} = 3\sqrt[5]{1+\frac{7}{243}}$, $\frac{7}{243} \in (-1,1)$

$f(x) = x^{\frac{1}{5}}$. Dále stačí dosadit do (Δ).

Průběhy funkcí

1. Základní věty: 1) $\exists f'(a)$ a f má lok. extrém v a $\Rightarrow f'(a) = 0$.

Ne naopak: $(x^3)'(0) = 3\cdot 0^2 = 0$  nemá v 0 extr.

2) $f \in C(a,b) \wedge \exists f'$: $f' > 0 \Rightarrow f$ roste, $f' \geq 0 \Rightarrow f$ neklesá

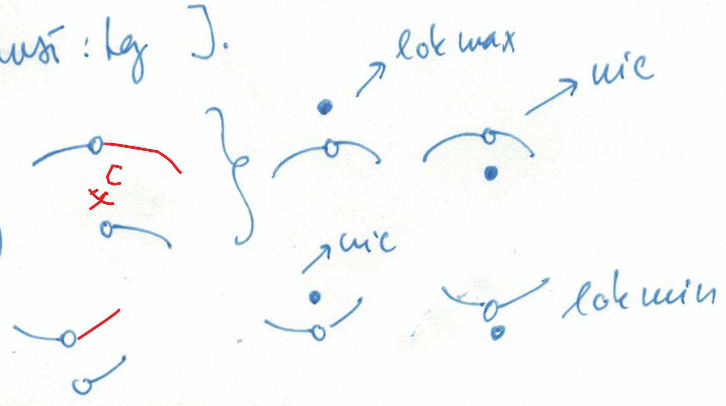
$f' < 0 \Rightarrow f$ klesá, $f' \leq 0 \Rightarrow f$ neroste

3) f spoj. na uzavř. $[a,b]$ má (globální) maximum i minimum

[Na otevřeném nemusí: tg].

4) a) f neklesá (a,c), f neroste (c,b)

b) naopak

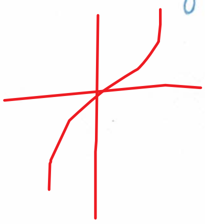


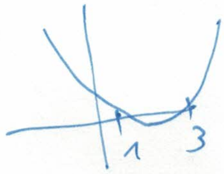
4) Používáme k vyšetřování ~~glob.~~ extr.

Př.: Nalezněte lok. extrémy $f(x)=x^3-6x^2+9x-4$, $x\in\mathbb{R}$.

$f'(x)=3x^2-12x+9=\underline{3(x-1)(x-3)}$

$f'=0 \Leftrightarrow x=1 \vee x=3$





	$(-\infty,1)$	$(1,3)$	$(3,\infty)$
$x-1$	$-$	$+$	$+$
$x-3$	$-$	$-$	$+$
f'	$+$	$-$	$+$

f

$x=1$ lok. max (dokonce ostré)

$x=3$ lok. min (dokonce ostré)

[$\lim\limits_{x\to\pm\infty} f(x)=\pm\infty$ (aritm. lim.) $\Rightarrow$ nejde o glob. extrémy]

Př.: Lok. extr. $f(x)=e^x\sin x$, $x\in\mathbb{R}$

$f'(x)=e^x\cos x+e^x\sin x=e^x(\sin x+\cos x)=0 \Leftrightarrow$

$\sin x=-\cos x$

$x_1=\frac{3}{4}\pi+2k\pi$

$x_2=\frac{7}{4}\pi+2k\pi$

$\Big\}\ \left(x_0=\frac{3}{4}\pi+k\pi\right)$

a) $\cos x\neq 0$

$\operatorname{tg} x=-1$

b) $\cos x=0$ $\quad\sin x=0$, pak ovšem $-\cos x=-1$

	$\left(-\frac{\pi}{4},\frac{3}{4}\pi\right)$	$\left(\frac{3}{4}\pi,\frac{7}{4}\pi\right)$	$\left(\frac{7}{4}\pi,\dots\right)$
f'	$+$	$-$	

$\frac{3}{4}\pi+2k\pi$ lok max

$\frac{7}{4}\pi+2k\pi$ lok min

[$\lim\limits_{x\to\frac{\pi}{2}+k\pi} f(x)=\lim\limits_{k\to\infty} e^{\frac{\pi}{2}+k\pi}\cdot 1=+\infty$; analog. $\lim\limits_{x\to-\frac{\pi}{2}+k\pi} \to-\infty$

glob. extr. $\nexists$]