Numerical solution of partial differential equations

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Course "Discontinuous Galerkin Method"
https://www2.karlin.mff.cuni.cz/~dolejsi/Vyuka/DGM.html

Partial differential equations

Why partial differential equations?

- many processes can be described (approximately) by PDEs
  - fluid dynamics, hydrology, heat and mass transfer, medicine, environmental protection, financial mathematics, etc.
  - these PDEs represent a mathematical description of physical, chemical, biological, etc. rules and/or laws
- some simplification usually necessary $\Longrightarrow$ model error
- these PDEs are usually too complicated for an exact solution

Numerical solution of PDEs

- we solve PDEs approximately (numerically)
- we define new simplified (finite dimensional, solvable) problem $\Longrightarrow$ discretization error

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Abstract problem described by PDEs

- let V be a functional space, we seek $u \in V$ such that

(EP) $$\mathcal{L}u = f$$

- $\mathcal{L}$ is a differential operator, f is a right-hand side,
- let solution of (EP) exists and is unique

Abstract numerical method

- let V_h be a space, $\dim(V_h) < \infty$, $V_h \subset V$ or $V_h \not\subset V$,
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Goals of the numerical solution of PDEs

Numerical analysis

- existence and uniqueness of u_h
- stability $\|u_h\| < \infty$
- convergence: $u_h \to u$ if dof $= \dim(V_h) \to \infty$
- estimate $\|u - u_h\|$ in terms of dof (a priori estimate)
- estimate $\|u - u_h\|$ based on u_h (a posteriori estimate)
- robustness: validity of previous items for large range of data

Numerical realization

- algorithm for fast evaluation of u_h (efficiency)
- stability of the method in the finite precision arithmetic
- adaptive strategies = adaptive changes of V_h

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Numerical method

Numerical method in practise

- finite sequence of mathematical operations
- output is the approximate solution u_h

Construction of a numerical method for the given PDE

- discretization (space, time)
- setting of arising algebraic systems (numerical quadratures)
- (iterative) solution of nonlinear algebraic systems
- solution of linear algebraic systems

Type of discretizations

finite difference method, finite element method, finite volume method, spectral method, wavelets method, etc.

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Choice of the numerical method

Which numerical method is the best one?

Depends on many aspects of the PDE considered

- physical background of the PDE
- expected regularity of the unknown exact solution
- presence of local phenomena
- outputs of interest
  - usual condition $\|u - u_h\| \leq TOL$ is not always practical
  - goal is the quantity of interest $J(u_h)$,

$$\Rightarrow \text{error: } |J(u) - J(u_h)| \leq TOL$$

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- conservation laws should be discretized by a conservative numerical method

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Which numerical method is the best one?

Depends on many aspects of the PDE considered

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Diffusion

- $\frac{\partial u}{\partial t} - \nabla \cdot (a(u)\nabla u) = g$
- parabolic (elliptic) equation
- quantity is spread in all directions
- influence is decreasing for increasing distance of the source

Convection

- $\frac{\partial u}{\partial t} - \nabla \cdot \vec{f}(u) = g$
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- quantity is spread only in the direction of convection $\vec{f}(u)$
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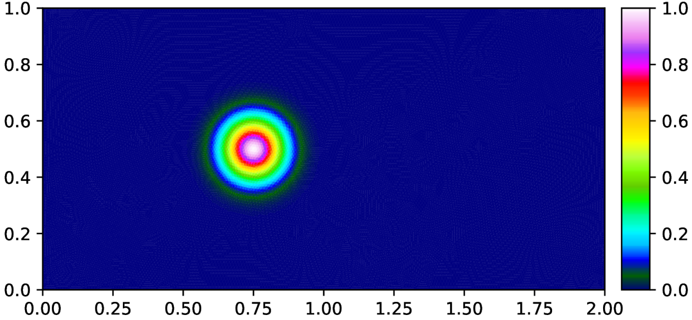
Examples of physical features (1)

Only diffusion

$$\frac{\partial u}{\partial t} - \varepsilon \Delta u = 0$$



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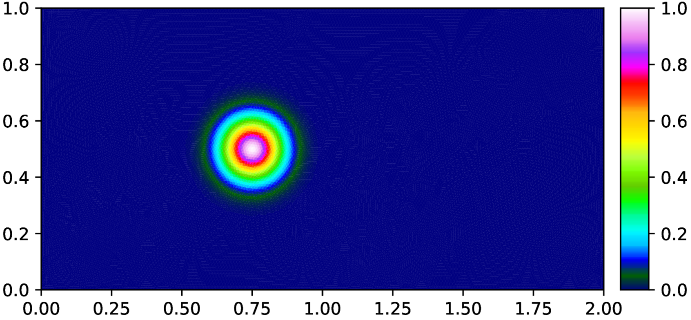
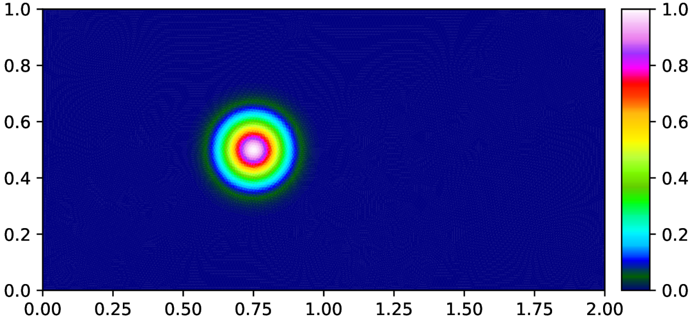


Examples of physical features (2)

Only convection:

$$\frac{\partial u}{\partial t} + \nabla \cdot (\vec{f}\, u) = 0, \quad \vec{f}(u) = (1,0)^T$$



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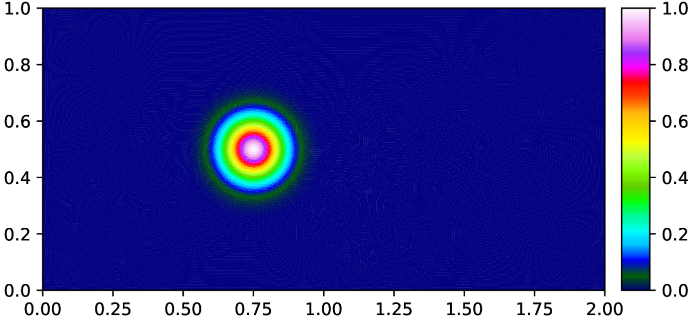

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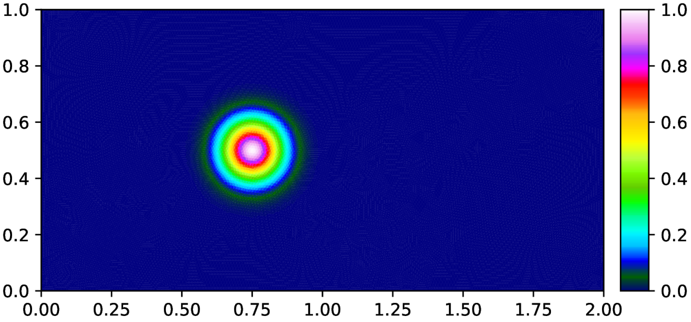

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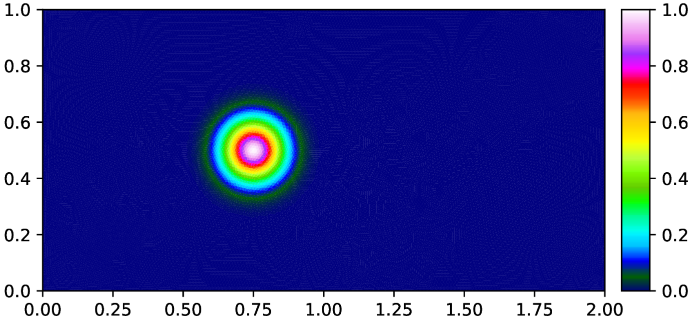

Elliptic and parabolic PDE vs. hyperbolic PDE

Cauchy problem

$$u(x,t) : \mathbb{R} \times (0,T) \to \mathbb{R} : \qquad \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = \varepsilon \frac{\partial^2 u}{\partial x^2}$$
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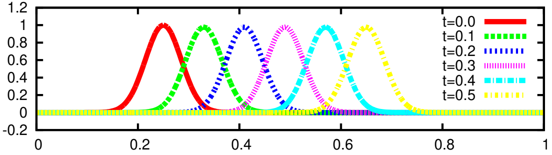

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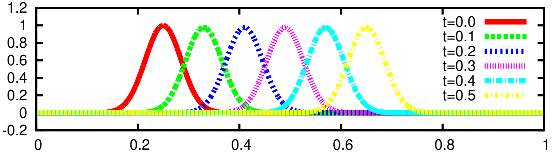

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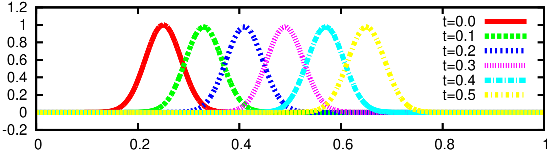

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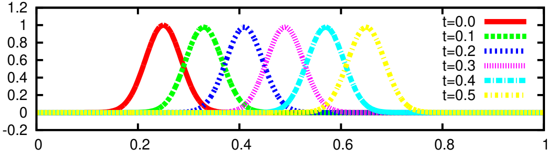

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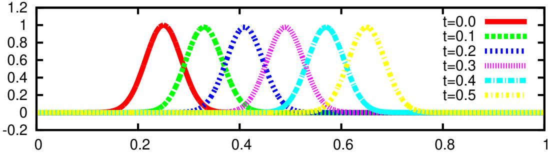

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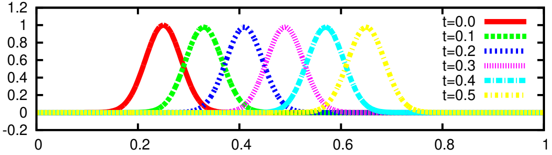

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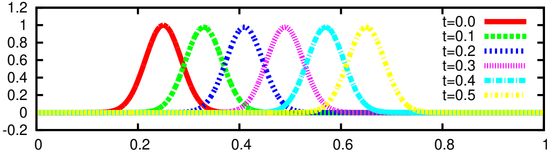

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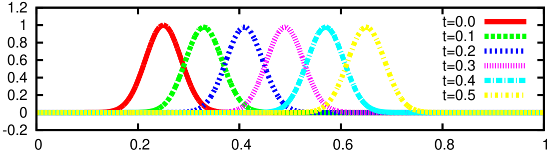

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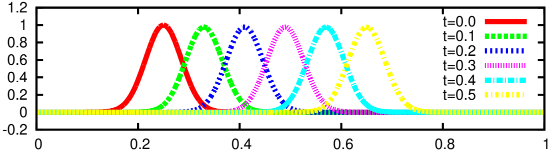

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Importance of the character of PDE

Why is important to know the previous properties?

- numerical solution is a kind of approximation
- many sources of inaccuracies:
  - discretization errors (finite dimensional approximation)
  - iterative errors (approximate solution of algebraic systems)
  - rounding errors (finite precision arithmetic)
- these inaccuracies are propagated by PDEs

Linear convection problem (no diffusion)

- exact solution: a simple propagation of the initial solution
- numerical solution: initial solution is propagated but smeared
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- numerical solution: initial solution is propagated but smeared
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Possible pitfalls

Effect of numerical diffusion

- zero diffusion does not exist in reality
- if numerical diffusion larger than physical one
  $\Longrightarrow$ numerical solution can be completely wrong
- e.g., numerical solution is steady whereas reality is unsteady

Effect of “finite h”

- we can prove that the proposed method is convergent
- approximate solution contains unphysical effects, e.g., spurious oscillations, negative temperature, etc.
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- No, it converges for $h \to 0$, the solution is bad for finite h

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$u : (0,1) \to \mathbb{R} : \quad -\varepsilon u'' + u' = f, \quad u(0) = u(1) = 0, \ \ \varepsilon > 0.$

solution has a steep gradient near $x = 1$ (boundary layer)

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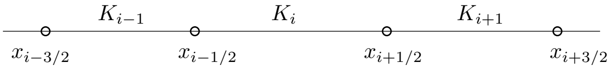

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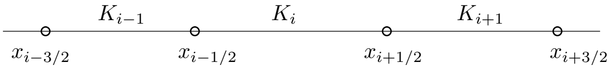

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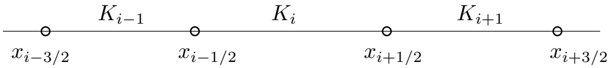

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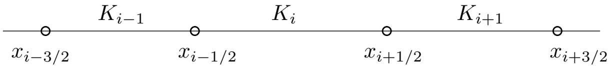

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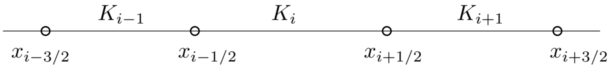

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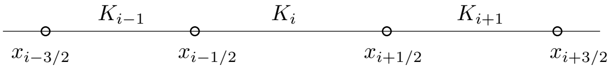

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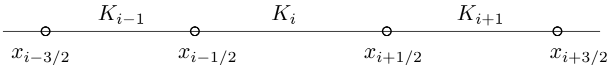

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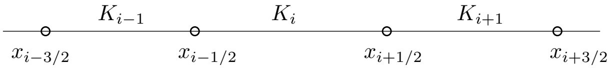

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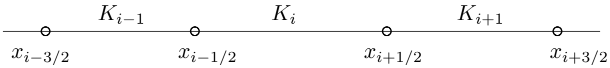

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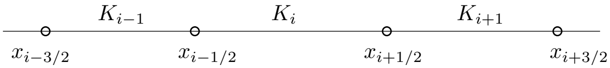
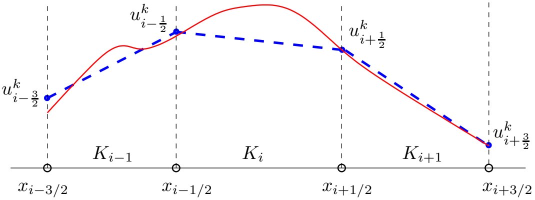


Finite element method



FEM solution

- $V_h = \{v_h \in C_0([0,1]);\ v_h|_{K_i} = P^1(K_i),\ i = 1,\dots,N\}$
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- discretization of convective term “does not respect physics”

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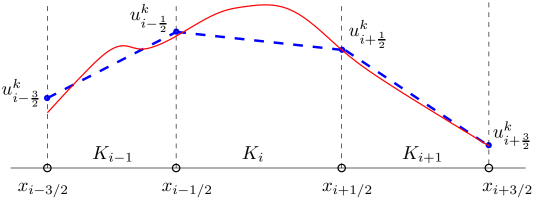

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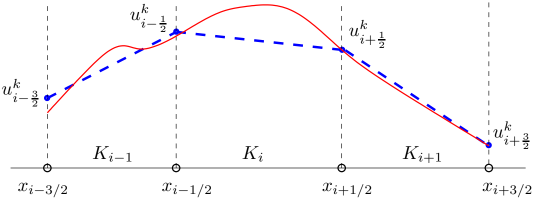

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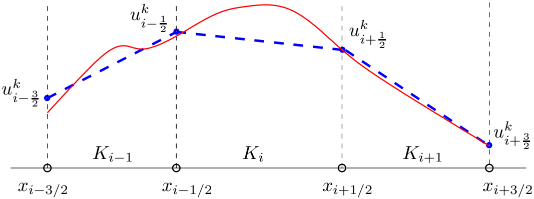

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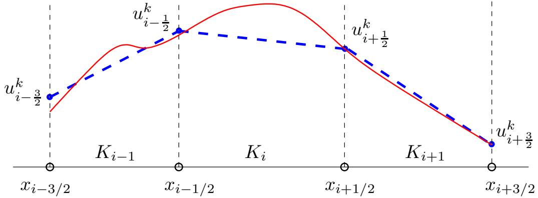

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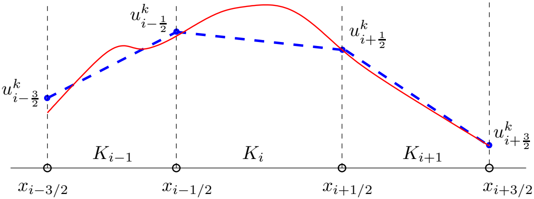

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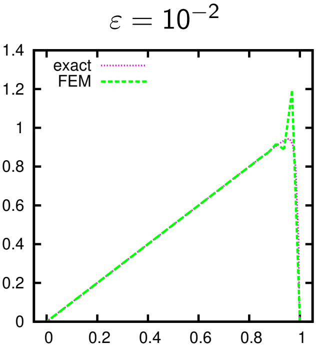

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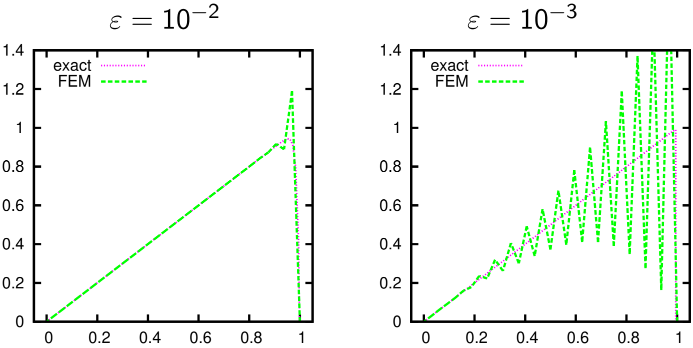

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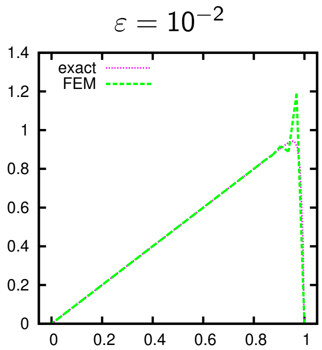

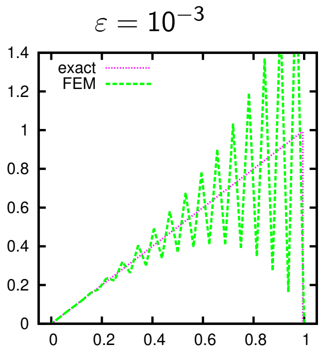

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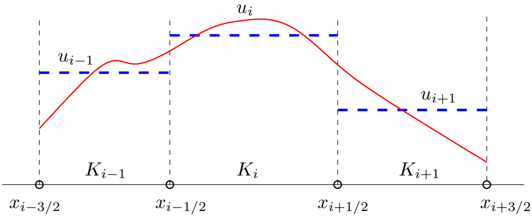

Piecewise constant approximation

- $V_h = \{v_h \in L^2([0,1]);\ v_h|_{K_i} = P^0(K_i),\ i = 1,\dots,N\}$
- we integrate $-\varepsilon u'' + au' = 1$ over K_i and use Gauss theorem

$$-\varepsilon[u'(\cdot)]_{x_{i-1/2}}^{x_{i+1/2}} + a[u(\cdot)]_{x_{i-1/2}}^{x_{i+1/2}} = |K_i|$$

- $u|_{x_{i+\frac{1}{2}}} = ??$ upwinding: $a > 0 \Rightarrow u|_{x_{i+\frac{1}{2}}} := u_i$

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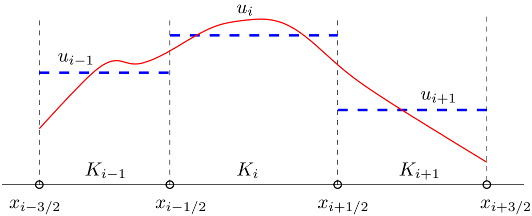

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- $V_h = \{v_h \in L^2([0,1]);\ v_h|_{K_i} = P^0(K_i),\ i = 1, \dots, N\}$
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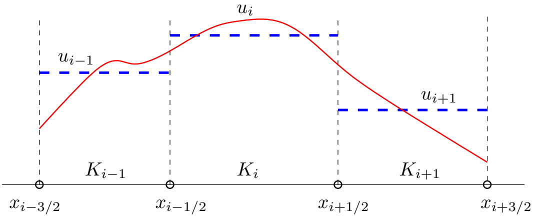

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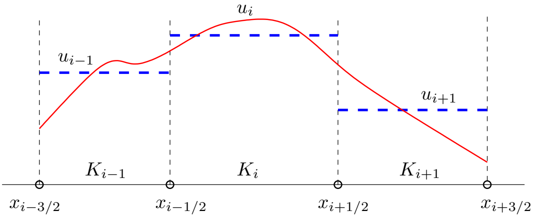

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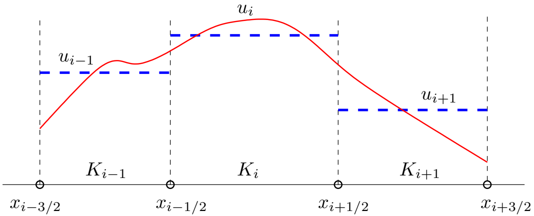

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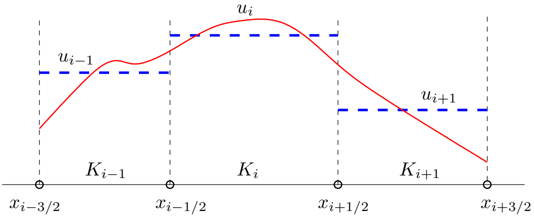

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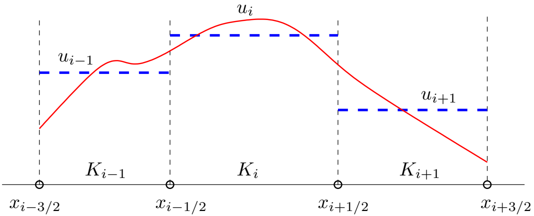

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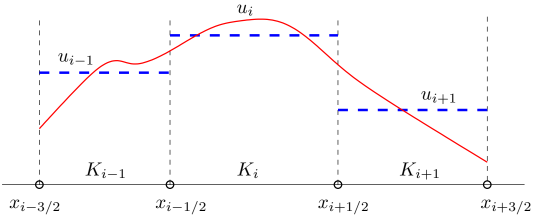

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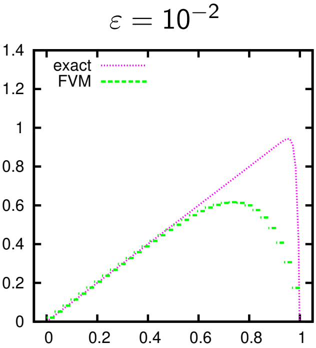

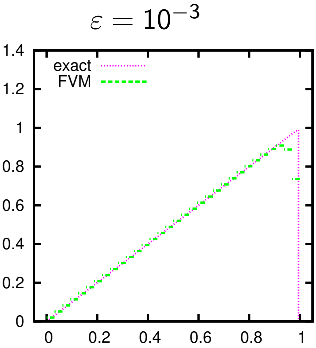

- Oscillations free approximation
- Low accuracy for larger ε
- A higher order reconstruction is a possible remedy

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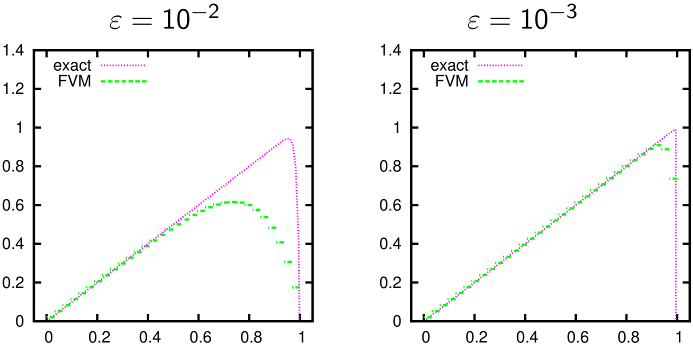

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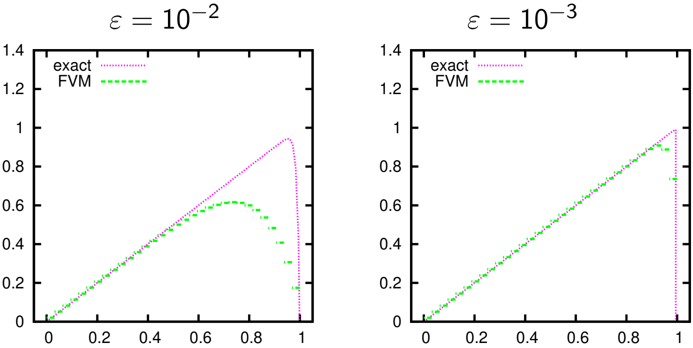

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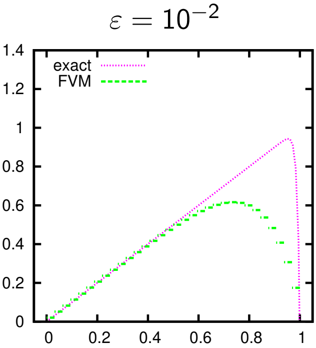

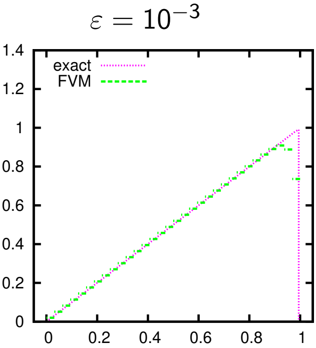

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Comparison of FEM and FVM

comparison of FEM and FVM for time-independent convective problem

Finite element method

- continuous approximation
- high order of accuracy
- many theoretical results
- fine for diffusive problems

Finite volume method

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Discontinuous Galerkin method

- piecewise polynomial discontinuous approximation
- theoretical justification
- higher freedom (adaptation, parallelization, etc.)

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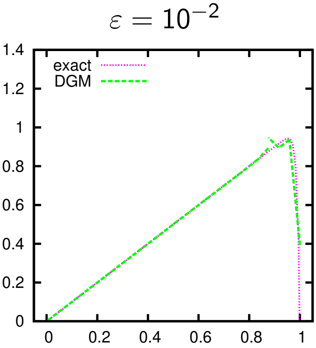

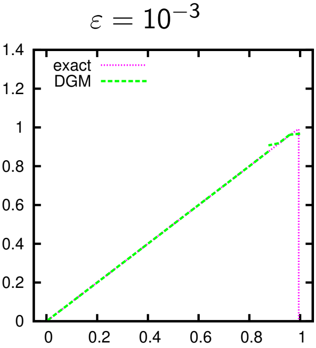

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- additional techniques (remedies) are possible

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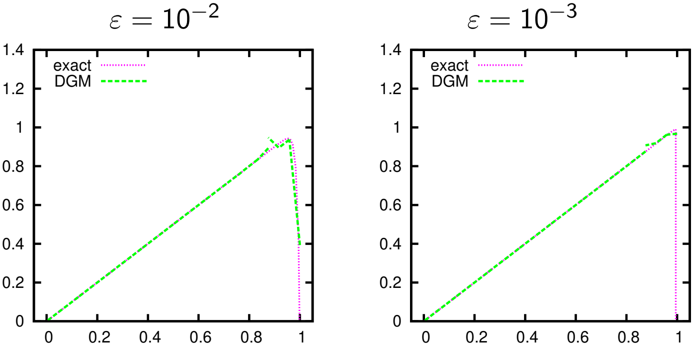

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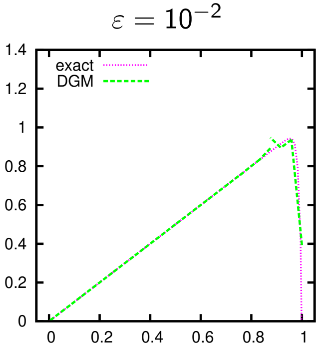

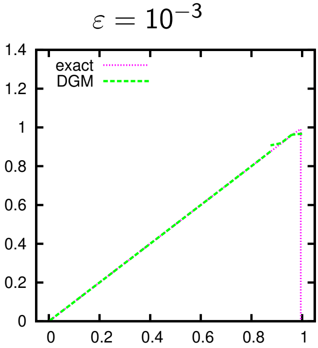

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Basic properties – positive

- efficient method for the numerical solution of various PDEs
- piecewise polynomial BUT discontinuous approximation
- suitable for very large range of problems
  - elliptic, parabolic, hyperbolic
  - linear, nonlinear, degenerate
- space-time DGMs are available
- flexibility in the mesh design
  - non-matching and non-uniform grids
  - anisotropic grids
  - varying polynomial approximation degrees
- (nice) block structure of arising algebraic systems
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- numerical analysis of the method is more complicated

Basic properties – practical

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- formulation of the method is more complicated
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Outline

- Abstract error analysis
- DGM for the Laplace problem: complete error analysis
- numerical approximation based on upwinding
- DGM for the nonlinear convection-diffusion equation
- DGM for time dependent problems
- DGM for compressible flow problems and other applications

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- 3 quizes during the semestr
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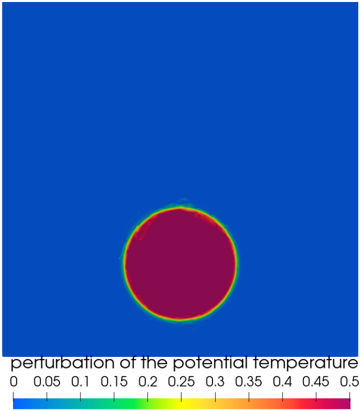
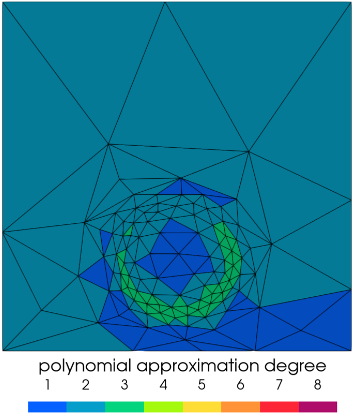
Modelling of mesoscale atmosphere by DGM

compressible Navier-Stokes equations with the gravity term





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